

19,21

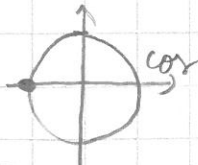
$$f(x) = \frac{5}{4 + 3 \underbrace{\cos 2x}_{-1 \leq \cos 2x \leq 1}}$$

$$\text{minimitejäen pienen arvo: } 4 + 3 \cdot (-1) = 1$$

$$\Rightarrow \text{minimitejä} \neq 0 \text{ aina } \Rightarrow x \in \mathbb{R}$$

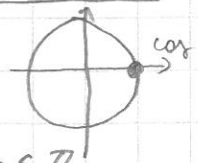
$$\text{suurin arvo: } \frac{5}{4 + 3 \cdot (-1)} = \frac{5}{1} = 5 \text{ kun } \cos 2x = -1$$

$$\Leftrightarrow 2x = \pi + M2\pi \quad | :2 \quad \Leftrightarrow x = \frac{\pi}{2} + M\pi, M \in \mathbb{Z}$$



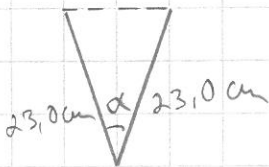
$$\text{pienin arvo: } \frac{5}{4 + 3 \cdot 1} = \frac{5}{7} \text{ kun } \cos 2x = 1$$

$$\Leftrightarrow 2x = M2\pi \quad | :2 \quad \Leftrightarrow x = M\pi, M \in \mathbb{Z}$$



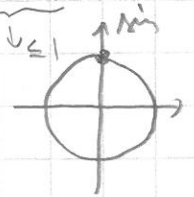
## 20. Sovellustehtäviä sin- ja kosinifunktioista

20.4



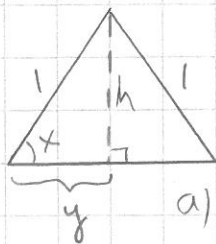
$$\text{pinta-ala: } A(\alpha) = \frac{1}{2} \cdot 23,0 \text{ cm} \cdot 23,0 \text{ cm} \cdot \underbrace{\sin \alpha}_{0 \leq \sin \alpha \leq 1}$$

$$A(\alpha) \text{ on suurin kun } \sin \alpha = 1 \Rightarrow \alpha = 90^\circ$$



$$\text{jälkeen tilavuus: } V = A \cdot h = \frac{1}{2} \cdot 2,30 \text{ dm} \cdot 2,30 \text{ dm} \cdot 30 \text{ dm} \\ = 79,35 \text{ dm}^3 = \underline{79 \text{ l}}$$

20.8



$$\begin{cases} \sin x = \frac{h}{1} = h \\ \cos x = \frac{y}{1} = y \end{cases}$$

$$\text{a) Pinta-ala: } A(x) = \frac{1}{2} \cdot 2y \cdot h = yh = \underline{\cos x \cdot \sin x}$$

$$\text{b) } A(x) = \cos x \cdot \sin x = \frac{1}{2} \cdot \frac{2 \sin x \cos x}{\sin 2x} = \frac{1}{2} \sin 2x$$

$$\text{suurin arvo: } \frac{1}{2} \cdot 1 = \frac{1}{2} \text{ kun } 2x = 90^\circ \quad | :2 \quad \Leftrightarrow x = 45^\circ$$

$$\text{Jälkeen kanta: } 2y = 2 \cos x = 2 \cdot \cos 45^\circ = 2 \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} \\ = \frac{2\sqrt{2}}{2} = \underline{\sqrt{2}}$$