16. Juurifunktion derivaatta

Kertausta :

$$a^{-n} = \frac{1}{a^n}$$

$$\left.\begin{aligned} a^{\frac{1}{n}} &= \sqrt[n]{a} \\ a^{\frac{k}{n}} &= \sqrt[n]{a^k} = \left(\sqrt[n]{a}\right)^k \end{aligned}\right\} \text{murtopotenssi}$$

$$D x^n = n x^{n-1}$$

$$D(f(x))^n = n\,(f(x))^{n-1} \cdot f'(x)$$

Esim. $D\sqrt{x} = D x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$

Yleisesti

$$\boxed{\begin{aligned} &D x^r = r x^{r-1}, \quad r \in \mathbb{R}, \; x > 0 \\ &D(f(x))^r = r\,(f(x))^{r-1} \cdot f'(x), \quad r \in \mathbb{R}, \; f(x) > 0 \end{aligned}}$$

16.2 a) $f(x) = \sqrt[3]{x^2-5} = (x^2-5)^{\frac{1}{3}}, \quad x \neq \pm\sqrt{5}$

$$f'(x) = \frac{1}{3}(x^2-5)^{\frac{1}{3}-1} \cdot 2x$$

$$\left[\; \frac{1}{3} - \overset{3)}{1} = \frac{1}{3} - \frac{3}{3} = \frac{1-3}{3} = \frac{-2}{3} = -\frac{2}{3} \;\right]$$

$$= \frac{1}{3}(x^2-5)^{-\frac{2}{3}} \cdot 2x = \frac{1}{3} \cdot \frac{1}{(x^2-5)^{\frac{2}{3}}} \cdot 2x$$

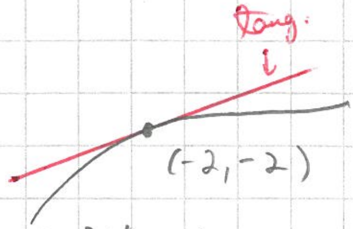
$$= \frac{2x}{3 \cdot \sqrt[3]{(x^2-5)^2}}$$

b) $g(x) = \sqrt{8x-1} = (8x-1)^{\frac{1}{2}}, \quad x > \frac{1}{8}$

$$g'(x) = \frac{1}{2}(8x-1)^{\frac{1}{2}-1} \cdot 8 = 4\,(8x-1)^{-\frac{1}{2}}$$

$$= 4 \cdot \frac{1}{(8x-1)^{\frac{1}{2}}} = \frac{4}{\sqrt{8x-1}}, \quad x > \frac{1}{8}$$

16.7 $y = x \cdot \sqrt[3]{x^2-3} = x \cdot (x^2-3)^{\frac{1}{3}}$



$x = -2: \; y = -2 \cdot \sqrt[3]{(-2)^2-3} = -2 \cdot \sqrt[3]{4-3} = -2 \cdot \sqrt[3]{1} = -2 \cdot 1 = -2$

$$\boxed{D(f \cdot g) = f'g + f \cdot g'}$$

$$y' = 1 \cdot (x^2-3)^{\frac{1}{3}} + x \cdot \frac{1}{3}(x^2-3)^{\frac{1}{3}-1} \cdot 2x$$

$$k_t = y'(-2) = \left((-2)^2-3\right)^{\frac{1}{3}} - 2 \cdot \frac{1}{3} \cdot \left((-2)^2-3\right)^{-\frac{2}{3}} \cdot 2 \cdot (-2)$$