

9.2

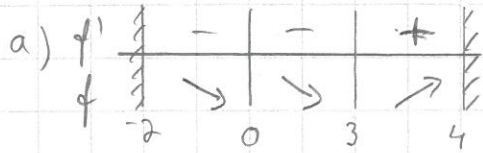
$$f(x) = x^4 - 4x^3$$

F ON JATKUVA JA DERIVOITUVA  
KOKO REAALILUKUJEN JOUKOSSA

$$f'(x) = 4x^3 - 12x^2 = 0$$

$$\Leftrightarrow 4x^2(x-3) = 0 \quad \Leftrightarrow 4x^2 = 0 \text{ tai } x-3 = 0$$

$$\Leftrightarrow x^2 = 0 \text{ tai } x=3 \quad \Leftrightarrow x=0 \text{ tai } x=3$$



$$f'(x) = \underbrace{4x^2}_{\geq 0} \underbrace{(x-3)}_{\begin{array}{c} + \\ - \\ + \end{array}}$$

pienin arvo:  $f(3) = 3^4 - 4 \cdot 3^3 = -27$

$$f(-2) = 48$$

$$f(4) = 0$$

$$\Rightarrow \text{suurin arvo: } \underline{48}$$

b) - pötearohdet:  $f(-2) = 48$

$$f(2) = -16$$

-  $f'$ :n 0-rohdet:  $f(0) = 0$

$$\Rightarrow \text{suurin arvo: } \underline{48}$$

$$\text{pienin arvo: } \underline{-16}$$

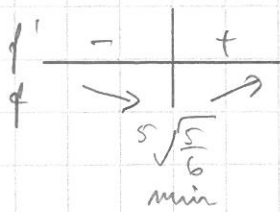
9.13 Väite  $x^6 + 7 > 5x$ ,  $x \in \mathbb{R}$

Tod. Väite  $\Leftrightarrow x^6 - 5x + 7 > 0$

$$f(x) = x^6 - 5x + 7, \quad f \text{ j\ddot{a}tt. j\ddot{a} deriv. R: N\ddot{a}e}$$

$$f'(x) = 6x^5 - 5 = 0 \quad \Leftrightarrow 6x^5 = 5 \quad | :6$$

$$\Leftrightarrow x^5 = \frac{5}{6} \quad | \sqrt[5]{\quad} \quad \Leftrightarrow x = \sqrt[5]{\frac{5}{6}} (\approx 0,964)$$



$$f'(0) = -5 < 0$$

$$f'(1) = 1 > 0$$

pienin arvo:  $f(\sqrt[5]{\frac{5}{6}}) \approx 2,98 > 0$   
 $\Rightarrow f(x) > 0$  aina  $\Rightarrow$  väite

Yleisesti Ep\ddot{a}yht\ddot{a}l\ddot{a}n  $g(x) > h(x)$  todistaminen:

$$g(x) > h(x) \quad \Leftrightarrow \underbrace{g(x) - h(x)}_{=f(x)} > 0$$

Fuukritaa  $f$ :n kullaa derivaatia aavulle

9.20  $f(t) = -0,0006t^4 + 0,02t^3 - 0,02t^2 + 0,88t + 22$

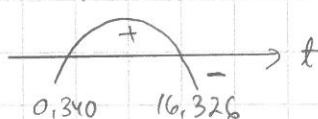
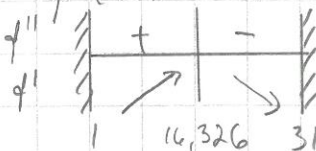
$f$  j\ddot{a}tt. j\ddot{a} deriv. v\ddot{a}l.  $[1, 31]$

$$f'(t) = -0,0024t^3 + 0,06t^2 - 0,04t + 0,88$$

$f'$  ilmaistaa miten nopeasti lumikerroksen p\ddot{a}k\ddot{a}n k\ddot{a}r\ddot{a}l\ddot{a} (pienenee)

$f'$  j\ddot{a}tt. j\ddot{a} deriv. v\ddot{a}l.  $[1, 31]$

$$f''(t) = -0,0072t^2 + 0,12t - 0,04 = 0 \quad \Leftrightarrow t = \begin{cases} 0,340 \\ 16,326 \end{cases} \quad (\text{v\ddot{a}l\ddot{a} k\ddot{a}r\ddot{a}e})$$



$$f'(1) = 0,8976$$

$$f'(16,326) = 5,776 \in \text{suurin arvo}$$

$$f'(31) = -14,158 \in \text{pienin}$$