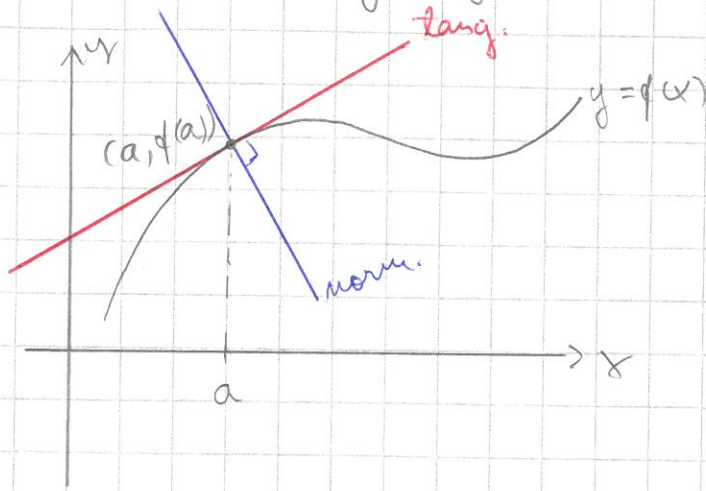


## 7. Tangentti ja normaali yhtälö



$k$  kulmokerroin :  $k$   
 suoran piste =  $(x_0, y_0)$   
 suoran yhtälö:  
 $y - y_0 = k(x - x_0)$

$$k_t = f'(a)$$

$$\Rightarrow \text{tangentti: } \underline{y - f(a) = f'(a)(x - a)}$$

$$k_t \cdot k_n = -1 \quad (\Leftrightarrow) \quad k_n = -\frac{1}{k_t} = -\frac{1}{f'(a)}$$

$$\Rightarrow \text{normaali: } \underline{y - f(a) = -\frac{1}{f'(a)}(x - a)}$$

7.5  $f(x) = -x^3 + 2x^2 + 1$

b)  $f(2) = -2^3 + 2 \cdot 2^2 + 1 = -8 + 8 + 1 = 1 \Rightarrow$  piste :  $(2, 1)$

$$f'(x) = -3x^2 + 4x$$

$$f'(2) = -3 \cdot 2^2 + 4 \cdot 2 = -12 + 8 = -4 = k_t$$

$$\text{tangentti: } y - 1 = -4(x - 2) \quad (\Leftrightarrow) \quad \underline{y = -4x + 9}$$

7.15  $y = -x^4 + 4x^2 - 2x = f(x)$

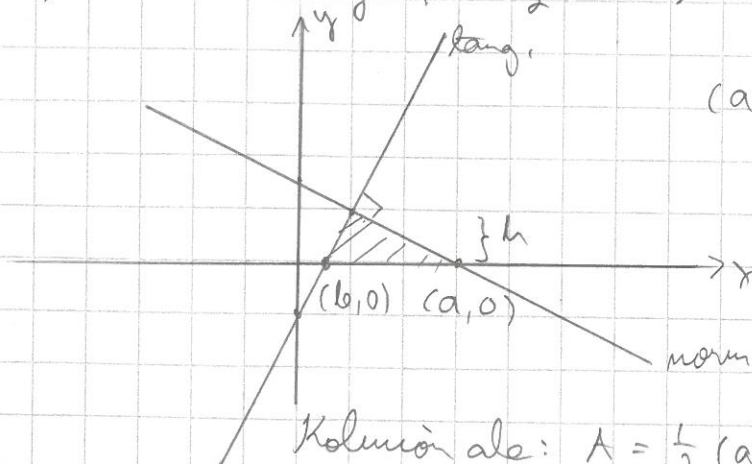
$$f(1) = -1^4 + 4 \cdot 1^2 - 2 \cdot 1 = -1 + 4 - 2 = 1 \Rightarrow \text{piste : } (1, 1)$$

$$f'(x) = -4x^3 + 8x - 2 \Rightarrow k_t = f'(1) = -4 \cdot 1^3 + 8 \cdot 1 - 2 = -4 + 8 - 2 = 2$$

$$\Rightarrow \text{tangentti: } y - 1 = 2(x - 1) \quad (\Leftrightarrow) \quad y = 2x - 1$$

$$k_n \cdot k_t = -1 \quad (\Leftrightarrow) \quad k_n = -\frac{1}{k_t} = -\frac{1}{2}$$

$$\Rightarrow \text{normaali: } y - 1 = -\frac{1}{2}(x - 1) \quad (\Leftrightarrow) \quad y = -\frac{1}{2}x + \frac{3}{2}$$



$(a, 0)$  on normaalilla :

$$0 = -\frac{1}{2}a + \frac{3}{2} \quad | \cdot 2$$

$$\Leftrightarrow a = 3$$

$(b, 0)$  on tangentilla :

$$0 = 2b - 1$$

$$\Leftrightarrow b = \frac{1}{2}$$

$$\text{Kulmion ala: } A = \frac{1}{2}(a - b) \cdot h = \frac{1}{2}(3 - \frac{1}{2}) \cdot 1$$

$$= \frac{5}{4}$$