5.21 a) g jatkuva kohdassa 0

$f(x) = x\,g(x)$

$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{x\,g(x) - \overbrace{0\cdot g(0)}^{=0}}{x} \lim_{x\to 0} \frac{\cancel{x}\,g(x)}{\cancel{x}}$$

$$= \lim_{x\to 0} g(x) = g(0)$$

g jatkuva kohdassa 0

b) $g(x) = |x|+1$

$$\lim_{x\to 0} g(x) = \lim_{x\to 0}(|x|+1) = 0+1 = 1 = g(0)$$

$\Rightarrow$ g on jatkuva kohdassa 0

$\Rightarrow$ voidaan soveltaa a-kohtaa

$$f'(0) = g(0) = \underline{1}$$

5.22 a) $f(x) = x^2$

$$f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{x^2-1^2}{x-1} \left(\frac{0}{0}\right) = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$= \lim_{x\to 1} (x+1) = 1+1 = \underline{2}$$

b) $$\lim_{h\to 0} \frac{f(1-h)-f(1)}{h}$$

$$\boxed{f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}}$$

$$= \lim_{h\to 0} \frac{f(1+(-h))-f(1)}{-h}\cdot(-1) = -1\cdot \underbrace{\lim_{-h\to 0} \frac{f(1+(-h))-f(1)}{-h}}_{f'(1)}$$

$$= -1\cdot f'(1) = -1\cdot 2 = \underline{-2}$$

c) $$\lim_{h\to 0} \frac{f(1+3h)-f(1)}{h} = \lim_{h\to 0} 3\cdot \frac{f(1+3h)-f(1)}{3h} = 3 \underbrace{\lim_{3h\to 0} \frac{f(1+3h)-f(1)}{3h}}_{f'(1)}$$

$$= 3\cdot f'(1) = 3\cdot 2 = \underline{6}$$

d) $$\lim_{h\to 0} \frac{f(1+h)-f(1-h)}{h} = \lim_{h\to 0} \frac{f(1+h)-f(1)+f(1)-f(1-h)}{h}$$

$$= \lim_{h\to 0}\left[\underbrace{\frac{f(1+h)-f(1)}{h}}_{\to f'(1)} - \underbrace{\frac{f(1-h)-f(1)}{h}}_{\to -f'(1)\ \text{b-kohta}}\right]$$

$$= f'(1) - (-f'(1)) = 2f'(1) = 2\cdot 2 = \underline{4}$$