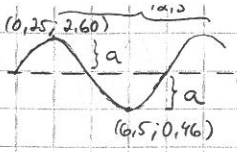
8.3 $f(x) = a\sin(bx+c)+d$



jakso : 12,5

suurin arvo klo 0.15 : $f(0,25) = 2,60$

pienin —"— klo 6.30 : $f(6,5) = 0,46$

keskiarvo : $d = \frac{2,60+0,46}{2} = 1,53$

amplitudi : $a = \frac{2,60-0,46}{2} = 1,07$

[TAI : $a = 2,60 - 1,53 = 1,07$ TAI : $a = 1,53 - 0,46 = 1,07$]

jakso : $\frac{2\pi}{b} = 12,5 \Leftrightarrow b = \frac{2\pi}{12,5} \approx 0,50265 \approx 0,503$

$f(0,25) = 1,07\sin(0,503\cdot 0,25 + c) + 1,53 = 2,60$

$\Leftrightarrow \sin(0,503\cdot 0,25 + c) = \frac{2,60-1,53}{1,07} = 1$

$\Rightarrow 0,503\cdot 0,25 + c = \frac{\pi}{2}$ (ens.) $\Leftrightarrow c \approx 1,45$

$\Rightarrow \underline{f(x) = 1,07\sin(0,503x + 1,45) + 1,53}$

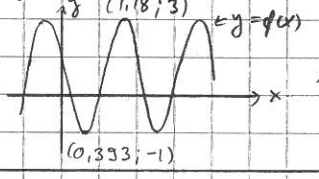
b) klo 23.15 : $f(23,25) = 2,11493 \approx \underline{2,1\,(m)}$

8.11 $f(x) = 2\sin(4x - \pi) + 1$

a) perusjakso : $\frac{2\pi}{4} = \frac{\pi}{2}$

b) suurin arvo : $2\cdot 1 + 1 = \underline{3}$

pienin arvo : $2\cdot(-1) + 1 = \underline{-1}$



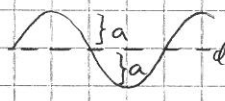
pienin arvo kun $x \approx 0,39$

suurin arvo kun $x \approx 1,18$

8.16 $f(x) = a\sin(bx+c) + d$

pienin arvo 21.6 : $f(172) = 9\frac{54}{60} = 9,9$

suurin —"— 21.12 : $f(355) = 14\frac{24}{60} = 14,4$



a) $d = \frac{9,9+14,4}{2} = 12,15$; $a = \frac{14,4-9,9}{2} = 2,25$

perusjakso : $\frac{2\pi}{b} = 2(355-172) = 366 \Leftrightarrow b = \frac{2\pi}{366} \approx 0,017167 \approx 0,0172$

$f(172) = 2,25\sin(0,0172\cdot 172 + c) + 12,15 = 9,9$

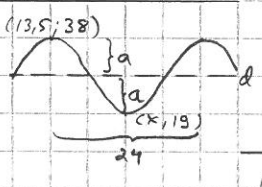
$\Leftrightarrow \sin(0,0172\cdot 172 + c) = \frac{9,9-12,15}{2,25} = -1$

$\Rightarrow 0,0172\cdot 172 + c = \frac{3\pi}{2} + n2\pi \overset{n=0}{\Rightarrow} c \approx 1,75$

$\Rightarrow \underline{f(x) = 2,25\sin(0,0172x + 1,75) + 12,2}$

b) 1.5 : $f(121) \approx 10,7185 \approx \underline{11\,(h)}$

8.18 $f(x) = a\sin(bx+c) + d$



suurin arvo 13.30 : $f(13,5) = 38$

pienin arvo : 19

perusjakso : 24

a) $d = \frac{38+19}{2} = 28,5$; $a = \frac{38-19}{2} = 9,5$

perusjakso : $\frac{2\pi}{b} = 24 \Leftrightarrow b = \frac{2\pi}{24} = \frac{\pi}{12} \approx 0,261799 \approx 0,262$

$f(13,5) = 9,5\sin(0,262\cdot 13,5 + c) + 28,5 = 38$

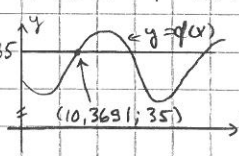
$\Leftrightarrow \sin(0,262\cdot 13,5 + c) = \frac{38-28,5}{9,5} = 1$

$\Leftrightarrow 0,262\cdot 13,5 + c = \frac{\pi}{2} + n2\pi \overset{n=0}{\Rightarrow} c \approx 4,31698 \approx 4,32$

$\Rightarrow \underline{f(x) = 9,50\sin(0,262x + 4,32) + 28,5}$

b) klo 19.45 : $f(19,75) \approx 27,8382 \approx \underline{28\,(°C)}$

c)



$0,3691\cdot 60 \approx 22,146$

$\Rightarrow$ klo 10.22

9.1 a) $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

b) $25^{\frac{1}{2}} = \sqrt{25} = 5$

c) $4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$

9.5 a) $\sqrt[3]{10^2} = 10^{\frac{2}{3}}$ b) $\sqrt{10^5} = 10^{\frac{5}{2}}$

c) $\sqrt[5]{100} = \sqrt[5]{10^2} = 10^{\frac{2}{5}}$ d) $\sqrt[12]{1000000} = \sqrt[12]{10^6} = 10^{\frac{6}{12}} = 1$

9.7 a) $\sqrt[6]{25} = \sqrt[6]{5^2} = 5^{\frac{2}{6}} = 5^{\frac{1}{3}} = \sqrt[3]{5}$

b) $\frac{5}{\sqrt{5}} = \frac{5^1}{5^{\frac{1}{2}}} = 5^{1-\frac{1}{2}} = 5^{\frac{1}{2}} = \sqrt{5}$ [TAI : $\frac{5}{\sqrt{5}} = \frac{\sqrt{5}\cdot\sqrt{5}}{\sqrt{5}} = \sqrt{5}$]

c) $\sqrt[6]{5}\cdot\sqrt[3]{5} = 5^{\frac{1}{6}}\cdot 5^{\frac{1}{3}} = 5^{\frac{1}{6}+\frac{1}{3}} = 5^{\frac{3}{6}} = 5^{\frac{1}{2}} = \sqrt{5}$

d) $(\sqrt[12]{5})^4 = 5^{\frac{4}{12}} = 5^{\frac{1}{3}} = \sqrt[3]{5}$

9.12 a) $\sqrt[5]{2} = 2^{\frac{1}{5}}$ b) $\sqrt[8]{3^5} = 3^{\frac{5}{8}}$

c) $\frac{1}{\sqrt{31}} = \frac{1}{31^{\frac{1}{2}}} = 31^{-\frac{1}{2}}$ d) $\frac{1}{\sqrt[6]{6^3}} = \frac{1}{6^{\frac{3}{6}}} = \frac{1}{6^{\frac{1}{2}}} = 6^{-\frac{1}{2}}$

9.14 a) $a\sqrt[3]{a} = a^1\cdot a^{\frac{1}{3}} = a^{1+\frac{1}{3}} = a^{\frac{4}{3}} \neq a^{\frac{7}{3}} \Rightarrow$ ei

b) $\sqrt[3]{a^7} = a^{\frac{7}{3}} \Rightarrow$ on

c) $\sqrt[7]{a^3} = a^{\frac{3}{7}} \Rightarrow$ ei $\quad a > 0$

d) $(\sqrt[3]{a})^7 = a^{\frac{7}{3}} \Rightarrow$ on

e) $a^2\sqrt{a} = a^2\cdot a^{\frac{1}{3}} = a^{2+\frac{1}{3}} = a^{\frac{7}{3}} \Rightarrow$ on

10.3 a) $0,5^x$; $0 < 0,5 < 1 \Rightarrow f$ aidosti vähenevä, piste $(1; 0,5) \Rightarrow$

b) $1,4^x$; $1,4 > 1 \Rightarrow f$ aidosti kasvava, piste $(1; 1,4) \Rightarrow \underline{3}$

c) $3,2^x$; $3,2 > 1 \Rightarrow$ —"— , piste $(1; 3,2) \Rightarrow \underline{2}$

10.4 $f(x) = (2-a)^x$

f aidosti kasvava $\Leftrightarrow 2-a > 1 \Leftrightarrow 1 > a \Leftrightarrow \underline{a < 1}$

10.6 $f(x) = 7500\cdot 7,9^x$

a) $f(2) = \underline{468075} \Rightarrow$ 2 h:in kuluttua tuon verran bakteereja

b) $f(0) = \underline{7500}$

c) 1 h:sta tulee 7,9 -kertaiseksi

d) $f(x) = 1000000 \Leftrightarrow x \approx 2,367 \approx \underline{2,4\,(h)}$

10.8 alussa : 1,0 (mg)

1 h:in kuluttua : $5\cdot 1,0$

2 —"— : $5^2\cdot 1,0$

⋮

a) x —"— : $m(x) = 5^x\cdot 1,0 = \underline{5^x\,(mg)}$

b) $m(4,5) \approx 1397,54 \approx \underline{1400\,(mg)}$

c) $y\cdot 5^2 = 1,0\,mg \;|:5^2 \Leftrightarrow y = \frac{1,0\,mg}{5^2} = \underline{0,04\,mg}$

Toisaalta : $m(-2) = 0,04\,(mg)$

10.9 a) $1,1^{2,5} < 1,1^{2,6}$ koska $1,1 > 1 \Rightarrow 1,1^x$ aidosti kasvava

b) $0,9^{2,5} > 0,9^{2,5}$ koska $0 < 0,9 < 1 \Rightarrow 0,9^x$ aidosti vähenevä

10.19 a) x^3 näyttää kasvavan nopeammin

b) $y = 1,4^x$ ja $y = x^3$ leikkaavat pisteessä $(30,46; 28265)$

c) $1,4^x > x^3$ kun $x = \underline{31} \Rightarrow$ saa

11.1 a) $\log_2 16 = 4$ $(2^4 = 16)$

b) $\log_2 8 = 3$ $(2^3 = 8)$

c) $\log_2 4 = 2$ $(2^2 = 4)$

d) $\log_2 2 = 1$ $(2^1 = 2)$

11.3 $9^{2021} = 10^x \Leftrightarrow x = \log_{10} 9^{2021} = 2021\log_{10} 9$

$\approx 1928,52411$