

5.1 a) $\sin x = 0,23 \approx \sin 0,2321$
 $\Rightarrow x \approx 0,23 + 2k\pi$ tai $x = \pi - 0,23 + 2k\pi = 2,91 + 2k\pi, k \in \mathbb{Z}$
 b) $5 \sin x = 4 \Rightarrow \sin x = \frac{4}{5} \approx \sin 0,9273$
 $\Rightarrow x \approx 0,93 + 2k\pi$ tai $x = \pi - 0,93 + 2k\pi = 2,21 + 2k\pi, k \in \mathbb{Z}$
 c) $3 \sin x - 6 = 0 \Rightarrow \sin x = 2$ ei ratka.

5.3 a) $\sin x = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3}$
 $\Rightarrow x = \frac{\pi}{3} + 2k\pi$ tai $x = \pi - \frac{\pi}{3} + 2k\pi = \frac{2\pi}{3} + 2k\pi, k \in \mathbb{Z}$
 b) $\sin 2x = \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4}$
 $\Rightarrow 2x = \frac{\pi}{4} + 2k\pi$ tai $2x = \pi - \frac{\pi}{4} + 2k\pi = \frac{3\pi}{4} + 2k\pi$ | :2
 $\Rightarrow x = \frac{\pi}{8} + k\pi$ tai $x = \frac{3\pi}{8} + k\pi, k \in \mathbb{Z}$

5.5 a) $\sin 5x = \sin x$
 $\Rightarrow 5x = x + 2k\pi$ tai $5x = \pi - x + 2k\pi$
 $\Rightarrow 4x = 2k\pi$ | :4 tai $6x = \pi + 2k\pi$ | :6
 $\Rightarrow x = M \frac{\pi}{2}$ tai $x = \frac{\pi}{6} + M \frac{\pi}{3}, M \in \mathbb{Z}$
 b) $x \in [0, \pi] : 0, \frac{\pi}{2}, 1, \frac{\pi}{2}, 2, \frac{\pi}{2}, \frac{\pi}{6} + 0, \frac{\pi}{3}, \frac{\pi}{6} + 1, \frac{\pi}{3}$
 $\Rightarrow 0, \frac{\pi}{2}, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

5.17 $\sin 2x = \frac{1}{2}$ KORJAUTTU:
 $2x = \frac{\pi}{6} + M \cdot 2\pi$ tai $2x = \frac{5\pi}{6} + M \cdot 2\pi$
 $x = \frac{\pi}{12} + M \cdot \pi$ tai $x = \frac{5\pi}{12} + M \cdot \pi, M \in \mathbb{Z}$

5.18 a) $\sin 5x = \sin(-x)$
 $\Rightarrow 5x = -x + 2k\pi$ tai $5x = \pi - (-x) + 2k\pi$
 $\Rightarrow 6x = 2k\pi$ | :6 tai $4x = \pi + 2k\pi$ | :4
 $\Rightarrow x = M \frac{\pi}{3}$ tai $x = \frac{\pi}{4} + M \frac{\pi}{2}, M \in \mathbb{Z}$
 b) $\sin 3x = \sin(-2x) = \sin(-2x)$
 $\Rightarrow 3x = -2x + 2k\pi$ tai $3x = \pi - (-2x) + 2k\pi$
 $\Rightarrow 5x = 2k\pi$ | :5 tai $x = \pi + 2k\pi$
 $\Rightarrow x = M \frac{2\pi}{5}$ tai $x = \pi + 2k\pi, M \in \mathbb{Z}$

5.21 $4 \sin^2 x = 3$ | :4 $\Rightarrow \sin^2 x = \frac{3}{4}$ | $\sqrt{\quad}$
 $\Rightarrow \sin x = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3}$ tai $\sin x = -\frac{\sqrt{3}}{2} = \sin(-\frac{\pi}{3})$
 $\Rightarrow x = \frac{\pi}{3} + 2k\pi$ tai $x = \pi - \frac{\pi}{3} + 2k\pi = \frac{2\pi}{3} + 2k\pi$
 tai $x = -\frac{\pi}{3} + 2k\pi$ tai $x = \pi - (-\frac{\pi}{3}) + 2k\pi = \frac{4\pi}{3} + 2k\pi, M \in \mathbb{Z}$

6.1 a) $\cos x = -0,70 \approx \cos 2,3462$
 $\Rightarrow x \approx \pm 2,35 + 2k\pi, k \in \mathbb{Z}$
 b) $4 \cos x - 3 = 0 \Rightarrow \cos x = \frac{3}{4} \approx \cos 0,7227$
 $\Rightarrow x = \pm 0,72 + 2k\pi, k \in \mathbb{Z}$
 c) $10 \cos x + 1 = 0 \Rightarrow \cos x = -\frac{1}{10} \approx \cos 1,6710$
 $\Rightarrow x \approx \pm 1,67 + 2k\pi, k \in \mathbb{Z}$

6.3 a) $\cos x = \frac{1}{2} = \cos \frac{\pi}{3}$
 $\Rightarrow x = \pm \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$
 b) $\cos(x - \frac{\pi}{6}) = \frac{1}{2} = \cos \frac{\pi}{3}$
 $\Rightarrow x - \frac{\pi}{6} = \frac{\pi}{3} + 2k\pi$ tai $x - \frac{\pi}{6} = -\frac{\pi}{3} + 2k\pi$
 $\Rightarrow x = \frac{\pi}{2} + 2k\pi$ tai $x = -\frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$

6.4 a) $\cos 4x = 1$
 $\Rightarrow 4x = 2k\pi$ | :4 $\Rightarrow x = M \frac{\pi}{2}, M \in \mathbb{Z}$
 b) $\cos(4x + \frac{\pi}{2}) = 1$
 $\Rightarrow 4x + \frac{\pi}{2} = 2k\pi \Rightarrow 4x = -\frac{\pi}{2} + 2k\pi$ | :4
 $\Rightarrow x = -\frac{\pi}{8} + M \frac{\pi}{2}, M \in \mathbb{Z}$

6.8 a) $2 \cos^2 x + \cos x - 1 = 0$
 Ann. $t = \cos x : 2t^2 + t - 1 = 0 \Rightarrow t = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$
 $\cos x = -1$ tai $\cos x = \frac{1}{2} = \cos \frac{\pi}{3}$
 $\Rightarrow x = \pi + 2k\pi$ tai $x = \pm \frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$
 b) $2 \sin^2 x + 3 \sin x - 2 = 0$
 Ann. $t = \sin x : 2t^2 + 3t - 2 = 0 \Rightarrow t = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$
 $\sin x = -2$ ei ratka tai $\sin x = \frac{1}{2} = \sin \frac{\pi}{6}$
 $\Rightarrow x = \frac{\pi}{6} + 2k\pi$ tai $x = \pi - \frac{\pi}{6} + 2k\pi = \frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z}$

6.16 a) $\cos \frac{x}{3} = \cos \frac{x}{6}$
 $\Rightarrow \frac{x}{3} = \frac{x}{6} + 2k\pi$ tai $\frac{x}{3} = -\frac{x}{6} + 2k\pi$ | :6
 $\Rightarrow 2x = x + 12k\pi$ tai $2x = -x + 12k\pi$
 $\Rightarrow x = 12k\pi$ tai $3x = 12k\pi$ | :3
 $\Rightarrow x = 4k\pi$ tai $x = 4k\pi, k \in \mathbb{Z}$
 b) $\sin \frac{x}{3} = \sin \frac{x}{2}$
 $\Rightarrow \frac{x}{3} = \frac{x}{2} + 2k\pi$ tai $\frac{x}{3} = \pi - \frac{x}{2} + 2k\pi$ | :6
 $\Rightarrow 2x = 3x + 12k\pi$ tai $2x = 6\pi - 3x + 12k\pi$
 $\Rightarrow -x = 12k\pi$ tai $5x = 6\pi + 12k\pi$ | :5
 $\Rightarrow x = -12k\pi$ tai $x = \frac{6\pi}{5} + M \frac{12\pi}{5}, M \in \mathbb{Z}$

6.15 $\cos 2x = \frac{1}{2}$ KORJAUS:
 $2x = \frac{\pi}{3}$ tai $2x = -\frac{\pi}{3}$ | :2
 $x = \frac{\pi}{6} + 2k\pi$ tai $x = -\frac{\pi}{6} + 2k\pi$
 $x = \pm \frac{\pi}{6} + 2k\pi, k \in \mathbb{Z}$

7.1 a) $f(x) = 4 \cos x$ arvojen väli: $[-4, 4]$, perusjakso: 2π
 b) $f(x) = \cos 4x$ arvojen väli: $[-1, 1]$, perusjakso: $\frac{2\pi}{4} = \frac{\pi}{2}$
 c) $f(x) = \cos \frac{1}{4} x$ arvojen väli: $[-1, 1]$, perusjakso: $\frac{2\pi}{\frac{1}{4}} = 8\pi$
 7.2 a) $f(x) = 2 \cos x, f(x) \in [-2, 2]$, perusjakso: $2\pi \Rightarrow 3$
 b) $g(x) = \cos \frac{1}{2} x, g(x) \in [-1, 1]$, perusjakso: $\frac{2\pi}{\frac{1}{2}} = 4\pi \Rightarrow 1$
 c) $h(x) = \cos 2x, h(x) \in [-1, 1], -1, -1, -1, \frac{2\pi}{2} = \pi \Rightarrow 2$

7.3 $f(x) = 2 + 4 \sin 3x$
 a) suurin arvo: $2 + 4 \cdot 1 = 6$
 pienin arvo: $2 + 4 \cdot (-1) = -2$ arvojen väli: $[-2, 6]$
 b) perusjakso: $\frac{2\pi}{3}$

7.5 $f(x) = a \sin x + d$
 suurin arvo: $a \cdot 1 + d = a + d = 1$
 pienin arvo: $a \cdot (-1) + d = -a + d = -3$
 $2d = -2 \Rightarrow d = -1$
 $\Rightarrow a = 1 - d = 1 - (-1) = 2$ (amplitudi)
 perusjakso: $5 \Rightarrow \frac{2\pi}{\omega} = 5 \Rightarrow \omega = \frac{2\pi}{5}$
 $\Rightarrow f(x) = 2 \sin(\frac{2\pi}{5} x) - 1$

7.20 $1 + 2 \sin x, x \in [0, \pi]$
 $0 \leq \sin x \leq 1$ suurin arvo: $1 + 2 \cdot 1 = 3$
 pienin arvo: $1 + 2 \cdot 0 = 1$

8.1 a) $f(x) = 7 \sin 6x$ suurin arvo: $7 \cdot 1 = 7$
 pienin arvo: $7 \cdot (-1) = -7$
 perusjakso: $\frac{2\pi}{6} = \frac{\pi}{3}$
 b) $f(x) = 5 \sin(8x - \pi) - 2$ suurin arvo: $5 \cdot 1 - 2 = 3$
 pienin arvo: $5 \cdot (-1) - 2 = -7$
 perusjakso: $\frac{2\pi}{8} = \frac{\pi}{4}$