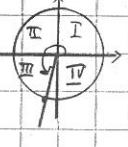
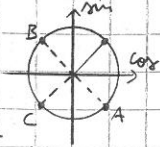


MAA5 Funktiot ja yhtälöt 2


1.1  a) $180^\circ < 263^\circ < 270^\circ \Rightarrow$ III-melji
 b) $-375^\circ = -360^\circ - 15^\circ \Rightarrow$ IV-melji
 c) $-1040^\circ = -2 \cdot 360^\circ - 320^\circ \Rightarrow$ I-melji

3.3 $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$
 a) A: $\sin(-\frac{\pi}{4}) = -\sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$
 b) B: $\sin \frac{3\pi}{4} = \sin(\pi - \frac{\pi}{4}) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$
 c) C: $\sin \frac{5\pi}{4} = \sin(\pi + \frac{\pi}{4}) = -\sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$


1.3 a) $\alpha = \frac{4,8 \text{ cm}}{3,7 \text{ cm}} \approx 1,2973 \approx 1,3$
 b) $\alpha = \frac{0,89 \text{ m}}{2,85 \text{ m}} \approx 0,312281 \approx 0,31$

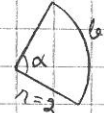
3.5 a) $\frac{\sin 10^\circ}{\cos 80^\circ} = \frac{\cos(90^\circ - 10^\circ)}{\cos 80^\circ} = \frac{\cos 80^\circ}{\cos 80^\circ} = 1$
 b) $\frac{\sin \frac{\pi}{8}}{\cos \frac{5\pi}{8}} = \frac{\cos(\frac{\pi}{2} - \frac{\pi}{8})}{\cos \frac{5\pi}{8}} = \frac{\cos \frac{3\pi}{8}}{-\cos(\pi - \frac{5\pi}{8})} = \frac{\cos \frac{3\pi}{8}}{-\cos \frac{3\pi}{8}} = -1$

1.5 a) $90^\circ = 90^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{2}$
 b) $45^\circ = 45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$
 c) $30^\circ = 30^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{6}$

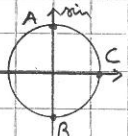
3.8 $\sin \alpha = \sin 30^\circ = \frac{1}{2}$
 a) $\alpha: 30^\circ + 360^\circ = 390^\circ, 30^\circ + 2 \cdot 360^\circ = 750^\circ$
 $30^\circ + 3 \cdot 360^\circ = 1110^\circ$
 b) $\alpha = 180 - 30^\circ = 150^\circ, 150^\circ + 360^\circ = 510^\circ$
 $150^\circ + 2 \cdot 360^\circ = 870^\circ$


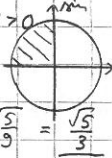
1.7 a) $\frac{7\pi}{6} = \frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = 210^\circ$ [TAI: $\frac{7\pi}{6} = \frac{7 \cdot 180^\circ}{6} = 210^\circ$
 b) $-\frac{5\pi}{9} = -\frac{5\pi}{9} \cdot \frac{180^\circ}{\pi} = -100^\circ$ $-\frac{5\pi}{9} = -\frac{5 \cdot 180^\circ}{9} = -100^\circ$
 c) $1,53 = 1,53 \cdot \frac{180^\circ}{\pi} \approx 87,6625^\circ \approx 87,7^\circ$

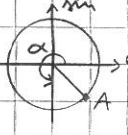
3.10 a) $\frac{25}{3}\pi = 6\frac{1}{3}\pi = \frac{\pi}{3} + 3 \cdot 2\pi \Rightarrow \frac{\pi}{3}$
 b) $-\frac{31}{6}\pi = -5\frac{1}{6}\pi = \frac{5\pi}{6} - 3 \cdot 2\pi \Rightarrow \frac{5\pi}{6}$
 [TAI: $\frac{25}{3}\pi \Rightarrow \frac{25}{3}\pi - 3 \cdot 2\pi = \frac{\pi}{3}$
 $-\frac{31}{6}\pi \Rightarrow -\frac{31}{6}\pi + 3 \cdot 2\pi = \frac{5\pi}{6}$]

1.13  $l = 3, r = 2, \alpha = \frac{l}{r} = \frac{3}{2}$
 $A = \frac{\alpha}{2\pi} \cdot \pi r^2 = \frac{\alpha}{2} r^2 = \frac{1}{2} \cdot \frac{3}{2} \cdot 2^2 = 3$

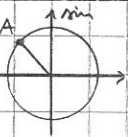
3.20 a) $1234^\circ = 154^\circ + 3 \cdot 360^\circ \Rightarrow 154^\circ$
 b) $18180^\circ = 180^\circ + 50 \cdot 360^\circ \Rightarrow 180^\circ$
 c) $-5840^\circ = -80^\circ - 16 \cdot 360^\circ \Rightarrow -80^\circ + 360^\circ = 280^\circ$
 [TAI: $1234^\circ \Rightarrow 1234^\circ - 3 \cdot 360^\circ = 154^\circ$
 $18180^\circ \Rightarrow 18180^\circ - 50 \cdot 360^\circ = 180^\circ$
 $-5840^\circ \Rightarrow -5840^\circ + 17 \cdot 360^\circ = 280^\circ$]

2.3  A: $\sin 90^\circ = 1$
 B: $\cos 270^\circ = 0$
 C: $\sin(-360^\circ) = 0$

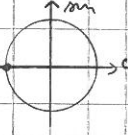
4.1 $\cos \alpha = -\frac{2}{3}, \frac{\pi}{2} \leq \alpha \leq \pi \Rightarrow$ II-melji $\Rightarrow \sin \alpha > 0$
 $\sin^2 \alpha + \cos^2 \alpha = 1 \Rightarrow \sin^2 \alpha = 1 - \cos^2 \alpha \sqrt{}$
 $\Rightarrow \sin \alpha = \pm \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - (-\frac{2}{3})^2} = \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$


2.5 $\alpha = \frac{7\pi}{4}$
 a) IV-melji
 b) $A = (\cos \alpha, \sin \alpha) = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$
 c) $\beta = \alpha + m2\pi = \frac{7\pi}{4} + m2\pi, m \in \mathbb{Z}$

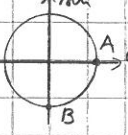
4.3 a) $\cos \alpha = -\frac{1}{2}$
 $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = \cos^2 \alpha - (1 - \cos^2 \alpha)$
 $= 2\cos^2 \alpha - 1 = 2 \cdot (-\frac{1}{2})^2 - 1 = 2 \cdot \frac{1}{4} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$
 b) $\sin \alpha = \frac{3}{5}$
 $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = (1 - \sin^2 \alpha) - \sin^2 \alpha$
 $= 1 - 2\sin^2 \alpha = 1 - 2 \cdot (\frac{3}{5})^2 = 1 - 2 \cdot \frac{9}{25} = 1 - \frac{18}{25} = \frac{7}{25}$

2.6  $A = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$
 a) II-melji
 b) $\beta = \frac{3\pi}{4} + m2\pi, m \in \mathbb{Z}$

4.5 $(\sin 2\alpha - \cos 2\alpha)^2 + \sin 4\alpha$
 $= \sin^2 2\alpha - 2\sin 2\alpha \cdot \cos 2\alpha + \cos^2 2\alpha + \sin(2 \cdot 2\alpha)$
 $= (\sin^2 2\alpha + \cos^2 2\alpha) - 2\sin 2\alpha \cdot \cos 2\alpha + 2\sin 2\alpha \cdot \cos 2\alpha$
 $= 1$

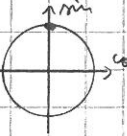
2.9  $A = (-1, 0) \Rightarrow \alpha = \pi + m2\pi, m \in \mathbb{Z}$

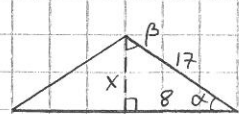
4.8 $(\sin \alpha + \cos \alpha)(\sin \alpha - \cos \alpha) = \sin^2 \alpha - \cos^2 \alpha$
 $= -(\cos^2 \alpha - \sin^2 \alpha) = -\cos 2\alpha$
 $\cos 2\alpha = -\frac{5}{27} \Rightarrow -(-\frac{5}{27}) = \frac{5}{27}$

2.13  A: $\cos 2\pi = 1$
 B: $\sin \frac{7\pi}{2} = -1$
 B: $\cos(-\frac{5\pi}{2}) = 0$

2.15 a) $\sin(2 \cos 70^\circ) = \sin(2 \cdot 0,34202) \approx 0,632$
 b) $\cos(\sin 23^\circ \cdot \cos 44^\circ) = \cos(-0,34622 \cdot 0,71934) \approx 0,820$

4.16 $10 \sin^2 \alpha - 3 \cos 2\alpha = 10 \sin^2 \alpha - 3(\cos^2 \alpha - \sin^2 \alpha)$
 $= 10 \sin^2 \alpha - 3(1 - 2\sin^2 \alpha) = 16 \sin^2 \alpha - 3$
 $\sin \alpha = -\frac{3}{4} \Rightarrow 16 \cdot (-\frac{3}{4})^2 - 3 = 16 \cdot \frac{9}{16} - 3 = 9 - 3 = 6$

3.1 a) $\sin \frac{9\pi}{2} = \sin(\frac{\pi}{2} + 2\pi) = \sin \frac{\pi}{2} = 1$
 b) $\cos(-\frac{13\pi}{2}) = \cos(\frac{\pi}{2} - 5 \cdot 2\pi) = \cos \frac{\pi}{2} = 0$
 c) $\sin 7\pi - \cos 8\pi = \sin(\pi + 3 \cdot 2\pi) - \cos(4 \cdot 2\pi)$
 $= \sin \pi - \cos 0 = 0 - 1 = -1$
 [TAI: $\sin \frac{9\pi}{2} = \sin(\frac{9\pi}{2} - 2\pi) = \sin \frac{\pi}{2} = 1$
 $\cos(-\frac{13\pi}{2}) = \cos(-9,5\pi) = \cos(-9,5\pi + 5 \cdot 2\pi) = \cos \frac{\pi}{2} = 0$
 $\sin 7\pi - \cos 8\pi = \sin(7\pi - 3 \cdot 2\pi) - \cos(8\pi - 4 \cdot 2\pi)$
 $= \sin \pi - \cos 0 = 0 - 1 = -1$]


4.17  $x^2 + 8^2 = 17^2 \Rightarrow x = 15$
 $\sin \beta = \frac{8}{17}, \cos \beta = \frac{x}{17} = \frac{15}{17}$
 $\sin 2\beta = 2 \sin \beta \cos \beta = 2 \cdot \frac{8}{17} \cdot \frac{15}{17} = \frac{240}{289}$