

$$\Leftrightarrow \log_2(4+2\sqrt{3}) = \log_2(2+\sqrt{3}) + 1 \quad \Leftrightarrow \log_2(2(2+\sqrt{3})) = \log_2(2+\sqrt{3}) + 1$$

$$\Leftrightarrow \log_2 2 + \log_2(2+\sqrt{3}) = \log_2(2+\sqrt{3}) + 1 \quad \text{! tori} \Rightarrow \text{v\ddot{a}ite}$$

5.1 $\sin x = \frac{1}{4}$, $\frac{\pi}{2} < x < \pi$  II melj. $\Rightarrow \cos x < 0$

$$\sin^2 x + \cos^2 x = 1 \quad \Leftrightarrow \cos^2 x = 1 - \sin^2 x \quad \sqrt{\quad} \quad \Leftrightarrow \cos x = \pm \sqrt{1 - \sin^2 x}$$

$$\cos x = -\sqrt{1 - \sin^2 x} = -\sqrt{1 - \left(\frac{1}{4}\right)^2} = -\sqrt{1 - \frac{1}{16}} = -\sqrt{\frac{15}{16}} = -\frac{\sqrt{15}}{4}$$

$$\sin 2x = 2 \sin x \cos x = 2 \cdot \frac{1}{4} \cdot \left(-\frac{\sqrt{15}}{4}\right) = -\frac{\sqrt{15}}{8}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 1 - 2 \cdot \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{8} = \frac{7}{8}$$

6. n. 1800 : 832 700
n. 1801 : $x \cdot 832 700$
n. 1802 : $x^2 \cdot 832 700$
:

a) n. 1850 : $x^{50} \cdot 832 700 = 1 636 900 \quad | : 832 700$
 $\Rightarrow x^{50} = \frac{1 636 900}{832 700} \quad | \sqrt[50]{\quad} \quad \Leftrightarrow x = \sqrt[50]{\frac{1 636 900}{832 700}} \approx 1,01361$
 $\Rightarrow$ kasvema reoodekse m. $x-1 \approx 0,01361 \approx 1,361\% \approx 1,4\%$

b) n. 2000 : $x^{200} \cdot 832 700 = 12,4343 \cdot 10^6 \approx 12,4 \cdot 10^6$

c) n. 1800 + t : $x^t \cdot 832 700 = 20 \cdot 10^6 \quad | : 832 700 > 0$
 $\Rightarrow x^t = \frac{20 \cdot 10^6}{832 700} \quad \Leftrightarrow t = \log_x \frac{16 369 000}{832 700} \approx 235,159$
 $\Rightarrow$ n. 2000 + 235 = 2035

7. $M = 3,2 + 0,67 \lg E$
 $\Leftrightarrow M - 3,2 = 0,67 \lg E \quad | : 0,67 \quad \Leftrightarrow \frac{M-3,2}{0,67} = \lg E \quad | 10^{(\quad)} \quad \Leftrightarrow E = 10^{\frac{M-3,2}{0,67}}$

a) $E = 10^{\frac{6,1-3,2}{0,67}} \approx 21 298,95 \text{ (MWh)} \Rightarrow$ m. 21 300 MWh
 $\frac{21 298,95 \text{ MWh}}{880 \text{ MWh/h}} \approx 24,2034 \text{ h} \approx \underline{24 \text{ h}}$

b) $E_2 = 10^{\frac{9,0-3,2}{0,67}} = 4,53645 \cdot 10^8 \Rightarrow \frac{E_2}{E} = \frac{4,53645 \cdot 10^8 \text{ MWh}}{21 298,95 \text{ MWh}} \approx 21 298,95$
 $\approx 21 300$ - bertainer

TAI: $\frac{E_2}{E} = \frac{10^{\frac{M_2-3,2}{0,67}}}{10^{\frac{M-3,2}{0,67}}} = 10^{\frac{M_2-3,2}{0,67} - \frac{M-3,2}{0,67}} = 10^{\frac{M_2-M}{0,67}} = 10^{\frac{9,0-6,1}{0,67}} \approx 21 298,95$

7.1 $\sqrt{a^3} \quad \frac{1}{a} \quad a^{\frac{3}{2}} \quad b \quad a^{-1} \quad a^{\frac{3}{2}} \cdot b \cdot a^{-1} = 1 \quad \Leftrightarrow b = \frac{1}{a^{\frac{3}{2}} \cdot a^{-1}} = \frac{1}{a^{\frac{1}{2}}} = \frac{1}{\sqrt{a}}$

$\sqrt{a} \quad \frac{1}{\sqrt{a}} \Rightarrow e \quad d \quad a^{\frac{5}{2}} \quad a^{-1} \cdot a^{\frac{5}{2}} \cdot c = 1 \quad \Leftrightarrow c = \frac{1}{a^{-1} \cdot a^{\frac{5}{2}}} = \frac{1}{a^{\frac{3}{2}}} = \frac{1}{a\sqrt{a}}$

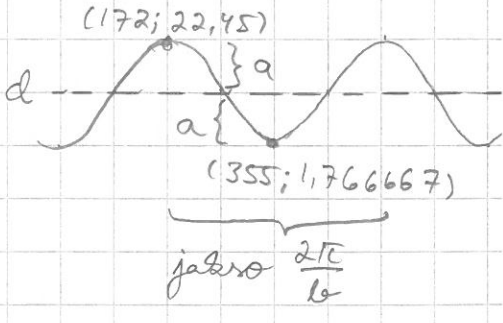
$b \cdot d \cdot a^{\frac{5}{2}} = 1 \quad \Leftrightarrow d = \frac{1}{a^{\frac{5}{2}} \cdot \frac{1}{a^{\frac{3}{2}}}} = \frac{1}{1} = 1$

$c \cdot d \cdot a^{\frac{5}{2}} = 1 \quad \Leftrightarrow c = \frac{1}{1 \cdot a^{\frac{5}{2}}} = \frac{1}{a^2 \sqrt{a}} \quad \text{Valt.} \quad \sqrt{a^3} \quad \frac{1}{\sqrt{a}} \quad \frac{1}{a}$

$f \cdot d \cdot a^{-1} = 1 \quad \Leftrightarrow f = \frac{1}{1 \cdot a^{-1}} = a \quad \frac{1}{a^2 \sqrt{a}} \quad 1 \quad \sqrt{a^5}$

$a \quad \sqrt{a} \quad \frac{1}{a\sqrt{a}}$

8. $f(t) = a \sin(b\omega t + c) + d$
 suurim ammu 21.6.: $f(172) = 22 + \frac{27}{60} = 22,45$
 pienim -11- 21.12.: $f(355) = 1 + \frac{46}{60} \approx 1,766667$



a) $d = \frac{22,45 + 1,766667}{2} \approx 12,10833$

$2a = 22,45 - 1,766667 \quad | : 2 \quad \Leftrightarrow a \approx 10,3417$
 TAI: $a = 22,45 - d = 10,3417$, TAI: $a = d - 1,766667 = 10,3417$

jaksu: $\frac{2\pi}{b} = 2(355 - 172) \quad \Rightarrow b = \frac{2\pi}{366} = \frac{\pi}{183} (= 0,01717)$

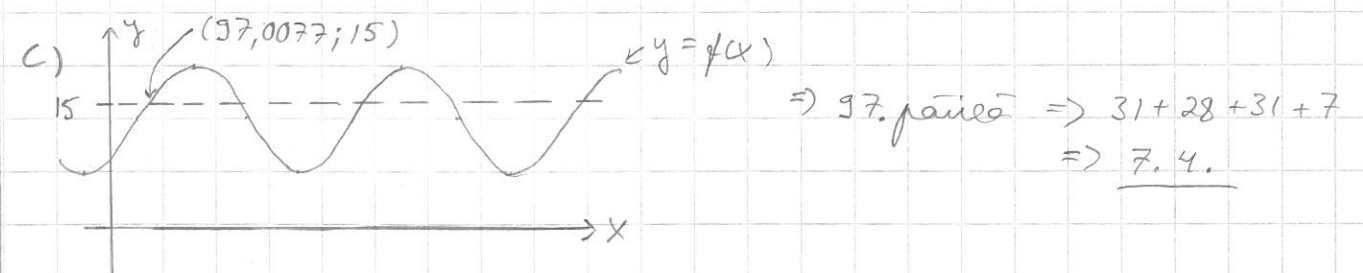
$f(172) = 10,3417 \sin\left(\frac{\pi}{183} \cdot 172 + c\right) + 12,10833 = 22,45 \quad | -12,10833 | : 10,3417$

$\Leftrightarrow \sin\left(\frac{\pi}{183} \cdot 172 + c\right) = 1 \quad \Leftrightarrow \frac{\pi}{183} \cdot 172 + c = \frac{\pi}{2} + M2\pi$

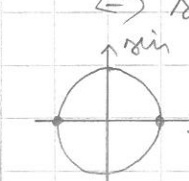
Valitaan $M = 0$: $c = \frac{\pi}{2} - \frac{\pi}{183} \cdot 172 = -\frac{161}{366}\pi (= -1,3820)$

$\Rightarrow f(t) = 10,3417 \sin\left(\frac{\pi}{183} \cdot t - \frac{161}{366}\pi\right) + 12,10833$

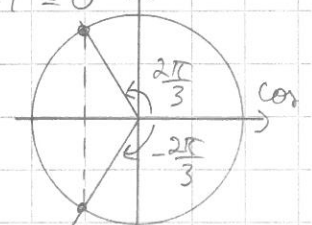
b) 5.1. : $f(15) = 2,154305 \Rightarrow \underline{2 \text{ h } 9 \text{ min}}$
 11.7. : $f(192) \approx 21,84643 \Rightarrow \underline{21 \text{ h } 51 \text{ min}}$



8.1 a) $2 \sin x \cos x + \sin x = 0$
 $\Leftrightarrow \sin x (2 \cos x + 1) = 0 \quad \Leftrightarrow \sin x = 0 \text{ tai } 2 \cos x + 1 = 0$



$x = M\pi \quad \text{tai} \quad \cos x = -\frac{1}{2} = \cos \frac{2\pi}{3}$
 $\Leftrightarrow x = \pm \frac{2\pi}{3} + M2\pi, M \in \mathbb{Z}$



TAI: $2 \sin x \cos x + \sin x = 0 \quad \Leftrightarrow \sin 2x + \sin x = 0 \quad \Leftrightarrow \sin 2x = -\sin x$
 $\Leftrightarrow \sin 2x = \sin(-x) \quad \Leftrightarrow 2x = -x + M2\pi \text{ tai } 2x = \pi - (-x) + M2\pi$
 $\Leftrightarrow 3x = M2\pi \text{ tai } x = \pi + M2\pi \quad \Leftrightarrow x = M\frac{2\pi}{3} \text{ tai } x = \pi + M2\pi, M \in \mathbb{Z}$