9. Murtopotenssi

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ \text{kpl}}$$

$$\boxed{\begin{aligned} a^0 &= 1 \\ a^{-n} &= \frac{1}{a^n} = \left(\frac{1}{a}\right)^n \end{aligned}} \quad a \neq 0$$

Potenssin laskusäännöt:

$$\boxed{\begin{aligned} (ab)^n &= a^n b^n \\ \left(\frac{a}{b}\right)^n &= \frac{a^n}{b^n} \\ a^n a^k &= a^{n+k} \\ \frac{a^n}{a^k} &= a^{n-k} \\ (a^n)^k &= a^{nk} \end{aligned}}$$

Esim. a) $4^{\frac{1}{2}} = 2$

b) $9^{\frac{1}{2}} = 3$

c) $16^{\frac{1}{2}} = 4$

d) $8^{\frac{1}{3}} = 2$

e) $27^{\frac{1}{3}} = 3$

Yleisesti

$$\boxed{\begin{aligned} a^{\frac{1}{n}} &= \sqrt[n]{a} \\ a^{\frac{k}{n}} &= \sqrt[n]{a^k} = (\sqrt[n]{a})^k \end{aligned}} \quad a > 0$$

potenssiin korotus

juurenotto

Tod. $(a^{\frac{1}{n}})^n = a^{\frac{1}{n} \cdot n} = a^1 = a \Rightarrow a^{\frac{1}{n}} = \sqrt[n]{a}$

$$a^{\frac{k}{n}} = \begin{cases} a^{\frac{1}{n} \cdot k} = (a^{\frac{1}{n}})^k = (\sqrt[n]{a})^k \\ a^{k \cdot \frac{1}{n}} = (a^k)^{\frac{1}{n}} = \sqrt[n]{a^k} \end{cases}$$

9.6 a) $3\sqrt{3} = 3^1 \cdot 3^{\frac{1}{2}} = 3^{1+\frac{1}{2}} = 3^{\frac{3}{2}}$

b) $9\sqrt{3} = 3^2 \cdot 3^{\frac{1}{2}} = 3^{2+\frac{1}{2}} = 3^{\frac{5}{2}}$

c) $\dfrac{1}{27\sqrt{3}} = \dfrac{1}{3^3 \cdot 3^{\frac{1}{2}}} = \dfrac{1}{3^{3+\frac{1}{2}}} = \dfrac{1}{3^{\frac{7}{2}}} = 3^{-\frac{7}{2}}$

d) $\dfrac{1}{\sqrt{3^7}} = \dfrac{1}{3^{\frac{7}{2}}} = 3^{-\frac{7}{2}}$