

luonn. dubiollaisen perusteiden on asettava!

2-kertaisen kulman sinin kaava on huolellisesti.

— 1 — kosinin — 1 — josta kaava.

Esim. $\sin \alpha = \frac{1}{4}$ ja $\frac{\pi}{2} < \alpha < \pi$. Löydä $\cos \alpha$, $\sin 2\alpha$ ja $\cos 2\alpha$

ratk. Perusteiden: $\sin^2 \alpha + \cos^2 \alpha = 1$

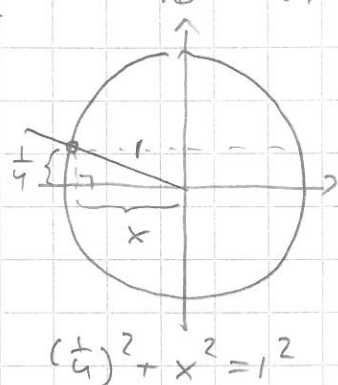
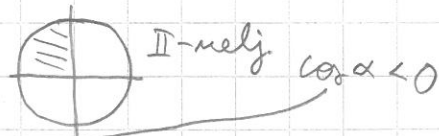
$$\Rightarrow \cos^2 \alpha = 1 - \sin^2 \alpha \quad | \sqrt{}$$

$$\Rightarrow \cos \alpha = \pm \sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \left(\frac{1}{4}\right)^2} = -\sqrt{1 - \frac{1}{16}} = -\sqrt{\frac{15}{16}}$$

$$= -\frac{\sqrt{15}}{4}$$

$$\begin{aligned} \sin 2\alpha &= 2 \sin \alpha \cos \alpha = 2 \cdot \frac{1}{4} \cdot \left(-\frac{\sqrt{15}}{4}\right) \\ &= -\frac{\sqrt{15}}{8} \end{aligned}$$

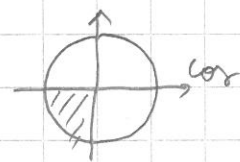
$$\begin{aligned} \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 1 - 2\sin^2 \alpha = 1 - 2 \cdot \left(\frac{1}{4}\right)^2 \\ &= 1 - 2 \cdot \frac{1}{16} = 1 - \frac{1}{8} = \frac{7}{8} \end{aligned}$$



$$x = -\frac{\sqrt{15}}{4}$$

4.4 $\sin \alpha = -\frac{2}{11}$

$\pi \leq \alpha \leq \frac{3\pi}{2}$ III-melj. $\Rightarrow \cos \alpha < 0$



Perusteiden: $\sin^2 \alpha + \cos^2 \alpha = 1 \quad \Rightarrow \cos^2 \alpha = 1 - \sin^2 \alpha \quad | \sqrt{}$

$$\Rightarrow \cos \alpha = \pm \sqrt{1 - \sin^2 \alpha} = -\sqrt{1 - \left(-\frac{2}{11}\right)^2} = -\sqrt{1 - \frac{4}{121}} = -\sqrt{\frac{117}{121}} = -\frac{\sqrt{117}}{11}$$

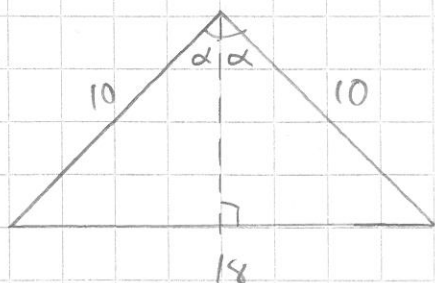
$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$= 2 \cdot \left(-\frac{2}{11}\right) \cdot \left(-\frac{\sqrt{117}}{11}\right) = \frac{4\sqrt{117}}{121} \quad \left(= \frac{12\sqrt{13}}{121} \right)$$

4.6

$$\begin{aligned} &(\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2 \\ &= (\sin^2 \alpha + 2 \sin \alpha \cos \alpha + \cos^2 \alpha) + (\sin^2 \alpha - 2 \sin \alpha \cos \alpha + \cos^2 \alpha) \quad \left[\begin{aligned} (a+b)^2 &= a^2 + 2ab + b^2 \\ (a-b)^2 &= a^2 - 2ab + b^2 \end{aligned} \right. \\ &= 2 \sin^2 \alpha + 2 \cos^2 \alpha = 2(\sin^2 \alpha + \cos^2 \alpha) = 2 \cdot 1 = 2 \end{aligned}$$

4.7



$$\sin \alpha = \frac{9}{10}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$= (1 - \sin^2 \alpha) - \sin^2 \alpha = 1 - 2\sin^2 \alpha$$

$$= 1 - 2 \cdot \left(\frac{9}{10}\right)^2 = 1 - 2 \cdot \frac{81}{100} = -\frac{31}{50}$$