$\cos(\bar{c},\bar{d}) = \dfrac{\bar{c}\cdot\bar{d}}{|\bar{c}||\bar{d}|} = \dfrac{-1\cdot(-5)+(-3)\cdot 5}{\sqrt{10}\sqrt{50}} = \dfrac{-10}{\sqrt{10}\sqrt{50}}$

$\Rightarrow \sphericalangle(\bar{c},\bar{d}) = \alpha \approx 116{,}57° \approx 117°$

$\Rightarrow$ lävistäjien välinen kulma: $\beta = 180° - \alpha \approx \underline{63°}$

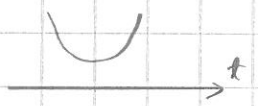
22.21 $\bar{a} = (t+1)\bar{i} + t\bar{j}$, $\bar{b} = 4\bar{i} + (t-3)\bar{j}$

$\bar{a}\cdot\bar{b} = (t+1)\cdot 4 + t\cdot(t-3) = 4t + 4 + t^2 - 3t$

$= t^2 + t + 4 > 0 \quad (0° < \alpha < 90° \Rightarrow \cos\alpha > 0)$

Vast. yhtälö: $t^2 + t + 4 = 0$, $D = 1^2 - 4\cdot 1\cdot 4 = -15 < 0$

$\Rightarrow$ ei ratk.



$\bar{a}\cdot\bar{b} = t^2 + t + 4 > 0$ aina

$\Rightarrow \sphericalangle(\bar{a},\bar{b})$ on aina terävä ($<90°$)

23.1 $A \xrightarrow{\quad C \quad} B$ $\quad A = (-1,3)$, $B = (7,-5)$

$\overline{AC} = \frac{1}{2}\overline{AB} = \frac{1}{2}(8\bar{i} - 8\bar{j}) = 4\bar{i} - 4\bar{j}$

$\Rightarrow C = (-1+4,\ 3-4) = \underline{(3,-1)}$

23.2 $A \xrightarrow{\quad M \quad} B$ $\quad A = (11,7)$, $M = (2,4)$

$\overline{AB} = 2\overline{AM} = 2(-9\bar{i} - 3\bar{j}) = -18\bar{i} - 6\bar{j}$

$\Rightarrow B = (11-18,\ 7-6) = \underline{(-7,1)}$

23.3 $y = -2x + 5 \Rightarrow k_1 = -2 \Rightarrow \bar{a} = \bar{i} - 2\bar{j}$ (suuntavektori)

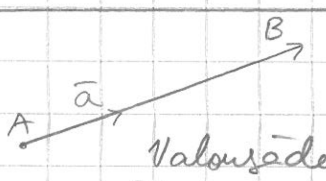
$y = 4x + 7 \Rightarrow k_2 = 4 \Rightarrow \bar{b} = \bar{i} + 4\bar{j}$ (— ,, —)

$\cos(\bar{a},\bar{b}) = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \dfrac{1\cdot 1 + (-2)\cdot 4}{\sqrt{1^2+(-2)^2}\sqrt{1^2+4^2}} = \dfrac{-7}{\sqrt{5}\sqrt{17}}$

$\Rightarrow \sphericalangle(\bar{a},\bar{b}) \approx 139{,}40° \approx 139°$

$\Rightarrow$ suorien välinen kulma ($\leq 90°$): $180° - 139° \approx \underline{41°}$

23.10



$A = (-14,0)$, $B = (26,16)$

$\bar{a} = 10\bar{i} + 4\bar{j}$

Valonsäde osuu B:hen $\Leftrightarrow \overline{AB} \uparrow\uparrow \bar{a}$

$\Leftrightarrow \overline{AB} = \underbrace{40}\bar{i} + \underbrace{16}\bar{j} = t(10\bar{i} + 4\bar{j}) = \underline{10t}\,\bar{i} + \underbrace{4t}\,\bar{j} \quad (t \geq 0)$

$\Rightarrow \begin{cases} 40 = 10t & \Leftrightarrow t = 4 \\ 16 = 4t & \Leftrightarrow t = 4 \end{cases} \Rightarrow \overline{AB} = 4\bar{a} \Rightarrow$ osuu

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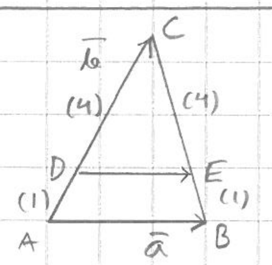
24.1



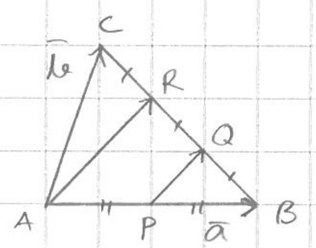
$\overline{AB} = \bar{a}$, $\overline{AC} = \bar{b}$

$\overline{DE} = \overline{DC} + \overline{CE} = \frac{4}{5}\bar{b} + \frac{4}{5}\overline{CB}$

$= \frac{4}{5}\bar{b} + \frac{4}{5}(-\bar{b} + \bar{a}) = \frac{4}{5}\bar{a}$

$\Rightarrow$ $\underline{DE \parallel AB \text{ ja } |DE| = \frac{4}{5}|AB|}$

24.2



$\overline{AB} = \bar{a}$, $\overline{AC} = \bar{b}$

$\overline{AR} = \overline{AC} + \overline{CR} = \bar{b} + \frac{1}{3}\overline{CB}$

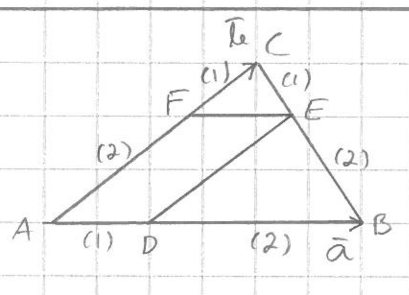
$= \bar{b} + \frac{1}{3}(-\bar{b} + \bar{a}) = \frac{1}{3}\bar{a} + \frac{2}{3}\bar{b}$

$\overline{PQ} = \overline{PB} + \overline{BQ} = \frac{1}{2}\bar{a} + \frac{1}{3}\overline{BC}$

$= \frac{1}{2}\bar{a} + \frac{1}{3}(-a + \bar{b}) = \frac{1}{6}\bar{a} + \frac{1}{3}\bar{b}$

$\Rightarrow \overline{AR} = 2\overline{PQ} \Rightarrow$ $\underline{AR \parallel PQ \text{ ja } |AR| = 2|PQ|}$

24.3



$\overline{AB} = \bar{a}$, $\overline{AC} = \bar{b}$

$\begin{cases} \overline{AF} = \frac{2}{3}\bar{b} \\ \overline{DE} = \overline{DB} + \overline{BE} = \frac{2}{3}\bar{a} + \frac{2}{3}\overline{BC} \end{cases}$

$= \frac{2}{3}\bar{a} + \frac{2}{3}(-a + \bar{b}) = \frac{2}{3}\bar{b}$

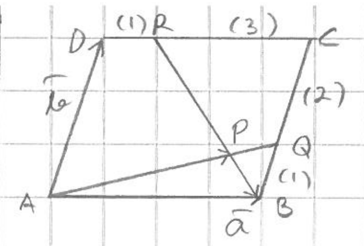
$\Rightarrow \overline{AF} = \overline{DE} \Rightarrow AF \parallel DE \quad (1)$

$\begin{cases} \overline{AD} = \frac{1}{3}\bar{a} \\ \overline{FE} = \overline{FC} + \overline{CE} = \frac{1}{3}\bar{b} + \frac{1}{3}\overline{CB} = \frac{1}{3}\bar{b} + \frac{1}{3}(-\bar{b} + \bar{a}) = \frac{1}{3}\bar{a} \end{cases}$

$\Rightarrow \overline{AD} = \overline{FE} \Rightarrow AD \parallel FE \quad (2)$

(1) ja (2) $\Rightarrow$ ADEF on suunnikas

24.12



$\overline{AB} = \bar{a}$, $\overline{AD} = \bar{b}$

$\begin{cases} \overline{AQ} = \bar{a} + \frac{1}{3}\bar{b} \\ \overline{AP} = x\overline{AQ} = x(\bar{a} + \frac{1}{3}\bar{b}) \end{cases}$

$\begin{cases} \overline{RB} = \overline{RC} + \overline{CB} = \frac{3}{4}\bar{a} - \bar{b} \\ \overline{PB} = y\overline{RB} = y(\frac{3}{4}\bar{a} - \bar{b}) \end{cases}$

Kierretään $A \to P \to B \to A$:

$\overline{AP} + \overline{PB} + \overline{BA} = \bar{0}$

$\Leftrightarrow x(\bar{a} + \frac{1}{3}\bar{b}) + y(\frac{3}{4}\bar{a} - \bar{b}) - \bar{a} = \bar{0}$

$\Leftrightarrow \underline{(x + \frac{3}{4}y - 1)}\,\bar{a} + \underbrace{(\frac{1}{3}x - y)}\,\bar{b} = \bar{0} = \underline{0}\cdot\bar{a} + \underbrace{0}\cdot\bar{b}$

$\bar{a} \nparallel \bar{b} \Rightarrow \begin{cases} x + \frac{3}{4}y - 1 = 0 \\ \frac{1}{3}x - y = 0 \quad |\cdot(-3) \end{cases}$

$\frac{15}{4}y - 1 = 0 \Leftrightarrow y = \frac{4}{15} \Rightarrow x = 3\cdot\frac{4}{15} = \frac{4}{5}$

a) $\dfrac{AP}{PQ} = \dfrac{\frac{4}{5}}{\frac{1}{5}} = \frac{4}{5}\cdot\frac{5}{1} = \underline{\frac{4}{1}}$ b) $\dfrac{BP}{PR} = \dfrac{\frac{4}{15}}{\frac{11}{15}} = \frac{4}{15}\cdot\frac{15}{11} = \underline{\frac{4}{11}}$