$C=(7,0)$: $(7+1)^2+(0-6)^2=100 \Leftrightarrow 64+36=100$ ✓

$\Rightarrow C$ on ympyrällä, $k_{AC}=\frac{\Delta y}{\Delta x}=\frac{0-6}{7-(-1)}=\frac{-6}{8}=-\frac{3}{4}$

$k_t\cdot k_{AC}=-1 \Leftrightarrow k_t=-\frac{1}{k_{AC}}=-\frac{1}{-\frac{3}{4}}=\frac{4}{3}$

Tangentti: $y-0=\frac{4}{3}(x-7) \quad |\cdot 3 \Leftrightarrow 4x-3y-28=0$

Leikkauspiste: $\begin{cases}3x+4y-71=0 & |\cdot 3\\ 4x-3y-28=0 & |\cdot 4\end{cases}$ yj.

$25x-325=0 \Leftrightarrow x=13$

$3\cdot 13+4y-71=0 \Leftrightarrow y=8 \Rightarrow \underline{(13,8)}$

13.14 $x^2+y^2-6x+4=0$

$\Leftrightarrow x^2-6x+y^2=-4 \quad |+3^2$

$\Leftrightarrow (x^2-2\cdot x\cdot 3+3^2)+y^2=-4+3^2$

$\Leftrightarrow (x-3)^2+y^2=5$, $k_p=A=(3,0)$, $r=\sqrt{5}$

$B=(3,\sqrt{5})$: $(3-3)^2+(\sqrt{5})^2=5 \Leftrightarrow 0+5=5$ ✓

$\Rightarrow B$ on ympyrällä $\Rightarrow 1$ tangentti

$k_{AB}=\frac{\Delta y}{\Delta x}=\frac{\sqrt{5}-0}{3-3}=\frac{\sqrt{5}}{0}$ ↯ $\Rightarrow$ sekä AB pystysuora

$\Rightarrow$ tangentti vaaksuora $\Rightarrow \underline{y=\sqrt{5}}$

13.20 $x^2+y^2+300x-400y+60\,900=0$

$\Leftrightarrow x^2+300x+y^2-400y=-60\,900 \quad |+150^2+200^2$

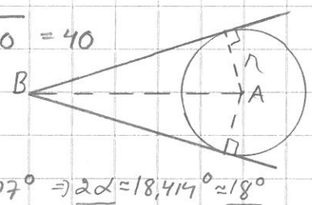
$\Leftrightarrow (x^2+2\cdot x\cdot 150+150^2)+(y^2-2\cdot y\cdot 200+200^2)=-60\,900+150^2+200^2$

$\Leftrightarrow (x+150)^2+(y-200)^2=1600$

$k_p=A=(-150,200)$, $r=\sqrt{1600}=40$

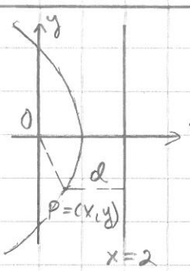
$B=(0,0)$

$|AB|=\sqrt{(-150-0)^2+(200-0)^2}=250$



$\sin\alpha=\frac{r}{|AB|}=\frac{40}{250} \Rightarrow \alpha\approx 9{,}207^\circ \Rightarrow \underline{2\alpha}=18{,}414^\circ\approx\underline{18^\circ}$

14.1



1° Olkoon $P=(x,y)$ mielivaltainen käyrän piste.

2° Yhtälö x:lle ja y:lle:

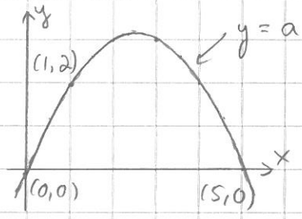
$|PO|=d$

$\Leftrightarrow \sqrt{(x-0)^2+(y-0)^2}=|x-2| \quad |()^2$ mol. puol. ≥ 0

3° Siirtymys: $\Leftrightarrow x^2+y^2=(x-2)^2$

$\Leftrightarrow x^2+y^2=x^2-4x+4 \Leftrightarrow \underline{x=-\frac{1}{4}y^2+1}$

14.5



$y=ax^2+bx+c$

$(0,0)$: $a\cdot 0^2+b\cdot 0+c=0$

$(1,2)$: $a\cdot 1^2+b\cdot 1+c=2$

$(5,0)$: $a\cdot 5^2+b\cdot 5+c=0$

$\Leftrightarrow \begin{cases}c=0 & (1)\\ a+b+c=2 & (2)\\ 25a+5b+c=0 & (3)\end{cases}$

$(3)-5\cdot(2)$: $20a-4c=-10 \Leftrightarrow 20a=-10 \Leftrightarrow a=-\frac{1}{2}$

(2): $b=2-a-c=2-(-\frac{1}{2})-0=\frac{5}{2}$

$\Rightarrow y=-\frac{1}{2}x^2+\frac{5}{2}x$

Suurin arvo saadaan 0-kohtien puolivälissä:

$x=2{,}5$: $y=-\frac{1}{2}\cdot 2{,}5^2+\frac{5}{2}\cdot 2{,}5=3{,}125\approx\underline{3{,}1\ (m)}$

⌈TAI: 0-kohdat: $x=0$ tai $x=5$

$\Rightarrow y=a(x-0)(x-5)=ax(x-5)$

$(1,2)$: $2=a\cdot 1\cdot(1-5)=-4a \Leftrightarrow a=-\frac{1}{2}$

$\Rightarrow y=-\frac{1}{2}x(x-5)=-\frac{1}{2}x^2+\frac{5}{2}x$ ⌋

14.8 $\begin{cases}4x+y+1=0\\ x^2-y+2=0\end{cases}$ $\Leftrightarrow y=-4x-1$ yj.

$\Rightarrow x^2-(-4x-1)+2=0$

$\Leftrightarrow x^2+4x+3=0 \Leftrightarrow x=\begin{cases}-1\\-3\end{cases}$

$x=-1$: $y=-4\cdot(-1)-1=3$

$x=-3$: $y=-4\cdot(-3)-1=11$

$\Rightarrow \underline{(-1,3) \text{ ja } (-3,11)}$

14.9 $x=ay^2+by+c$ (annettu sivulla)

$\begin{cases}(-4,1): & a\cdot 1^2+b\cdot 1+c=-4\\ (0,3): & a\cdot 3^2+b\cdot 3+c=0\\ (5,4): & a\cdot 4^2+b\cdot 4+c=5\end{cases} \Leftrightarrow \begin{cases}a+b+c=-4 & (1)\\ 9a+3b+c=0 & (2)\\ 16a+4b+c=5 & (3)\end{cases}$

$(2)-(1)$: $\begin{cases}8a+2b=4\\ 7a+b=5 & |\cdot(-2)\end{cases}$

$(3)-(2)$: $-6a=-6 \Leftrightarrow a=1$

$\Rightarrow b=5-7\cdot 1=-2$, (1): $c=-4-1-(-2)=-3$

$\Rightarrow \underline{x=y^2-2y-3}$

14.13 a) $y=2x-x^2+4$: $\underline{3}$ ($a=-1<0 \Rightarrow$ aukeaa alaspäin)

b) $y^2=-x+y+1 \Leftrightarrow x=-y^2+y+1$: $\underline{1}$ ($a=-1<0 \Rightarrow$ vasemmalle)

c) $y=x^2+6x+10$: $\underline{2}$ ($a=1>0 \Rightarrow$ ylöspäin)

d) $-x=-y^2+y-4 \Leftrightarrow x=y^2-y+4$: $\underline{4}$ ($a=1 \Rightarrow$ oikealle)

15.1 $(x_0,y_0)=(-1,2)$

a) $y-2=a(x-(-1))^2 \Leftrightarrow y-2=a(x+1)^2$

$(0,0)$: $0-2=a(0+1)^2 \Leftrightarrow -2=a$

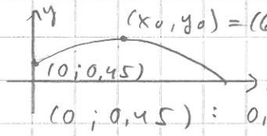
$\Rightarrow y-2=-2(x+1)^2 \Leftrightarrow y=-2(x^2+2x+1)+2 \Leftrightarrow \underline{y=-2x^2-4x}$

b) $x-(-1)=a(y-2)^2$

$(0,0)$: $0+1=a(0-2)^2 \Leftrightarrow 1=4a \Leftrightarrow a=\frac{1}{4}$

$\Rightarrow x+1=\frac{1}{4}(y-2)^2 \Leftrightarrow x=\frac{1}{4}(y^2-4y+4)-1 \Leftrightarrow \underline{x=\frac{1}{4}y^2-y}$

15.3



$(x_0,y_0)=(6{,}4;0{,}95)$

$y-0{,}95=a(x-6{,}4)^2$

$(0;0{,}45)$: $0{,}45-0{,}95=a(0-6{,}4)^2 \Leftrightarrow a=-\frac{0{,}5}{6{,}4^2}$

$\Rightarrow y=-\frac{0{,}5}{6{,}4^2}(x-6{,}4)^2+0{,}95=0 \quad |\cdot(-\frac{6{,}4^2}{0{,}5})$

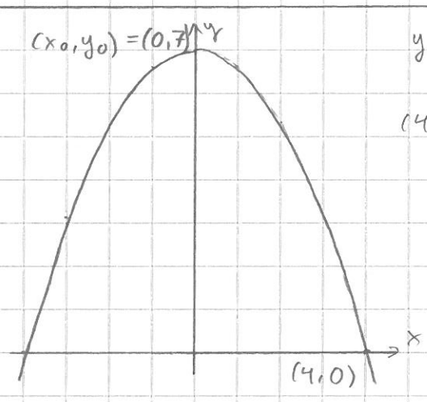
$\Leftrightarrow (x-6{,}4)^2=-0{,}95\cdot(-\frac{6{,}4^2}{0{,}5}) \quad |\sqrt{\ }$

$\Leftrightarrow x-6{,}4=\pm\sqrt{0{,}95\cdot\frac{6{,}4^2}{0{,}5}} \Leftrightarrow x=6{,}4\pm\sqrt{0{,}95\cdot\frac{6{,}4^2}{0{,}5}}$

$\approx\begin{cases}15{,}2218\\ -2{,}4218\end{cases}$

Vast. n. 15 m

15.15



$y-7=a(x-0)^2$

$\Leftrightarrow y=ax^2+7$

$(4,0)$: $0=a\cdot 4^2+7$

$\Leftrightarrow a=-\frac{7}{16}$

$\Rightarrow y=-\frac{7}{16}x^2+7$

a) Eriksen kuljetus korkeinta kohtaa:

$x=\frac{3{,}6}{2}=1{,}8$

$y(1{,}8)=-\frac{7}{16}\cdot 1{,}8^2+7=5{,}583$

$<5{,}6$

$\Rightarrow$ ei mahu

b) $y(2{,}6)=-\frac{7}{16}\cdot 2{,}6^2+7=4{,}043>3{,}9 \Rightarrow$ mahtuu

⌈TAI: $y=ax^2+bx+c$

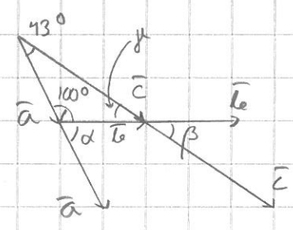
$\begin{cases}(4,0): & a\cdot 4^2+b\cdot 4+c=0\\ (-4,0): & a\cdot(-4)^2+b\cdot(-4)+c=0\\ (0,7): & a\cdot 0^2+b\cdot 0+c=7\end{cases} \Leftrightarrow \begin{cases}16a+4b+c=0 & (1)\\ 16a-4b+c=0 & (2)\\ c=7 & (3)\end{cases}$

$(1)+(2)$: $32a+2c=0 \Leftrightarrow a=-\frac{2}{32}\cdot c=-\frac{7}{16}$

$(1)-(2)$: $8a=0 \Leftrightarrow b=0$

$\Rightarrow y=-\frac{7}{16}x^2+7$ ⌋

16.5



a) $\measuredangle(\bar{a},\bar{c})=\underline{43^\circ}$

b) $\measuredangle(\bar{a},\bar{b})=\alpha=180^\circ-100^\circ=\underline{80^\circ}$

c) $\measuredangle(\bar{b},\bar{c})=\beta=\gamma=180^\circ-100^\circ-43^\circ$

$=\underline{37^\circ}$