12.2

$$\begin{cases} x-2y-2=0 & \Leftrightarrow x=2y+2 \\ x^2+y^2+4x-6y-5=0 & \text{sij.} \end{cases}$$

$\Rightarrow (2y+2)^2+y^2+4(2y+2)-6y-5=0$

$\Leftrightarrow 4y^2+8y+4+y^2+8y+8-6y-5=0$

$\Leftrightarrow 5y^2+10y+7=0$, $D=10^2-4\cdot5\cdot7=-40<0 \Rightarrow$ ei rat.

$\Rightarrow$ ei leikkauspisteitä

12.3

$$\begin{cases} x^2+y^2-5=0 & |\cdot(-1) \\ x^2+y^2+8y-13=0 & \text{sij.} \end{cases}$$

$8y-8=0 \Leftrightarrow y=1$

$\Rightarrow x^2+1^2-5=0 \Leftrightarrow x^2=4 \quad |\sqrt{\ } \Leftrightarrow x=\pm2$

Vast. $(-2,1)$ ja $(2,1)$

12.5

b) $x^2+y^2-2ax-2ay+4a-2=0$

$$\begin{cases} a=0: & x^2+y^2-2=0 \\ a=1: & x^2+y^2-2x-2y+4-2=0 \quad |\cdot(-1) \end{cases} \text{sij.}$$

$2x+2y-4=0 \Leftrightarrow y=-x+2$

$\Rightarrow x^2+(-x+2)^2-2=0 \Leftrightarrow x^2+x^2-4x+4-2=0$

$\Leftrightarrow 2x^2-4x+2=0 \quad |:2 \Leftrightarrow x^2-2x+1=0$

$\Leftrightarrow (x-1)^2=0 \quad |\sqrt{\ } \Leftrightarrow x-1=0 \Leftrightarrow x=1$

$y=-1+2=1 \Rightarrow$ piste $(1,1)$

Tark. $(1,1)$: $1^2+1^2-2a\cdot1-2a\cdot1+4a-2=0 \Leftrightarrow 0=0$ ✓

$\Rightarrow$ kaikki ympyrät kulkevat pisteen $(1,1)$ kautta

12.6

$$\begin{cases} x-y+3=0 & \Leftrightarrow y=x+3 \\ x^2+y^2-8x-2y+1=0 & \text{sij.} \end{cases}$$

$\Rightarrow x^2+(x+3)^2-8x-2(x+3)-1=0$

$\Leftrightarrow x^2+x^2+6x+9-8x-2x-6-1=0$

$\Leftrightarrow 2x^2-4x+2=0 \quad |:2 \Leftrightarrow x^2-2x+1=0$

$\Leftrightarrow (x-1)^2=0 \quad |\sqrt{\ } \Leftrightarrow x-1=0 \Leftrightarrow x=1 \Rightarrow y=1+3=4$

$\Rightarrow$ vain 1 yhteinen piste $(1,4) \Rightarrow$ suora on tangentti

⌈ TAI: $x^2+y^2-8x-2y-1=0$

$\Leftrightarrow (x-4)^2+(y-1)^2=18$, kp $=(4,1)$, $r=\sqrt{18}=\sqrt{9\cdot2}=3\sqrt{2}$

kp:n etäisyys suorasta:

$$d=\frac{|4-1+3|}{\sqrt{1^2+(-1)^2}}=\frac{\sqrt{2}/6}{\sqrt{2}}=\frac{6\sqrt{2}}{2}=3\sqrt{2}=r$$

$\Rightarrow$ suora on tangentti ⌋

12.17

$(x+3)^2+(y-1)^2=20$, kp $=A=(-3,1)$, $r_1=\sqrt{20}\approx4{,}4$

$(x-3)^2+(y+2)^2=1$, kp $=B=(3,-2)$, $r_2=\sqrt{1}=1$

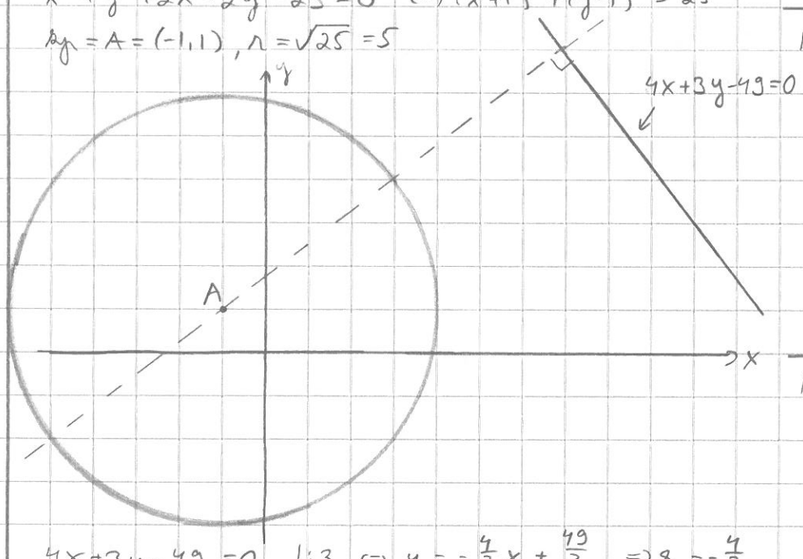
$|AB|=\sqrt{(-3-3)^2+(1-(-2))^2}=\sqrt{45}\approx6{,}51>r_1+r_2$

$\Rightarrow$ ympyrät eivät leikkaa

12.19

$x^2+y^2+2x-2y-23=0 \Leftrightarrow (x+1)^2+(y-1)^2=25$

kp $=A=(-1,1)$, $r=\sqrt{25}=5$



$4x+3y-49=0 \quad |:3 \Leftrightarrow y=-\frac{4}{3}x+\frac{49}{3} \Rightarrow k_1=-\frac{4}{3}$

$k_1\cdot k_2=-1 \Leftrightarrow k_2=-\frac{1}{k_1}=-\frac{1}{-\frac{4}{3}}=\frac{3}{4}$

normaali: $y-1=\frac{3}{4}(x-(-1)) \Leftrightarrow y=\frac{3}{4}x+\frac{7}{4}$

Leikkauspisteet: $\begin{cases} y=\frac{3}{4}x+\frac{7}{4} & \text{sij.} \\ x^2+y^2+2x-2y-23=0 \end{cases}$

$\Rightarrow x^2+\left(\frac{3}{4}x+\frac{7}{4}\right)^2+2x-2\left(\frac{3}{4}x+\frac{7}{4}\right)-23=0$

$\Leftrightarrow x^2+\frac{9}{16}x^2+\frac{21}{8}x+\frac{49}{16}+2x-\frac{3}{2}x-\frac{7}{2}-23=0 \quad |\cdot16$

$\Leftrightarrow 25x^2+50x-375=0 \Leftrightarrow x=-5$ tai $x=3$

kuva: lähimmässä pisteessä $x=3$

$\Rightarrow y=\frac{3}{4}\cdot3+\frac{7}{4}=4 \Rightarrow$ piste $(3,4)$

13.1

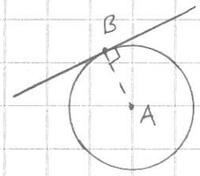
$(x+6)^2+(y-2)^2=20$, kp $=A=(-6,2)$, $r=\sqrt{20}=2\sqrt{5}$

$B=(-8,6)$: $(-8+6)^2+(6-2)^2=20 \Leftrightarrow 4+16=20$ ✓

$\Rightarrow$ piste B on ympyrällä $\Rightarrow$ 1 tangentti

$k_{AB}=\frac{\Delta y}{\Delta x}=\frac{2-6}{-6-(-8)}=\frac{-4}{2}=-2$

$k_t\cdot k_{AB}=-1 \Leftrightarrow k_t=-\frac{1}{k_{AB}}=-\frac{1}{-2}=\frac{1}{2}$



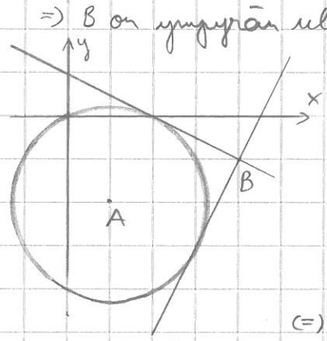
tangentti: $y-6=\frac{1}{2}(x-(-8)) \Leftrightarrow y=\frac{1}{2}x+10$

13.3

b) $(x-1)^2+(y+2)^2=5$, kp $=A=(1,-2)$, $r=\sqrt{5}$

$B=(4,-1)$: $|AB|=\sqrt{(4-1)^2+(-1-(-2))^2}=\sqrt{10}>r$

$\Rightarrow$ B on ympyrän ulkopuolella $\Rightarrow$ 2 tangenttia



tangentti: $y-(-1)=k(x-4)$

$\Leftrightarrow kx-y-4k-1=0$

kp:n etäisyys tangentista = säde:

$$d=\frac{|k\cdot1-(-2)-4k-1|}{\sqrt{k^2+(-1)^2}}=\frac{|1-3k|}{\sqrt{k^2+1}}=\sqrt{5} \quad |\cdot\sqrt{k^2+1}$$

$\Leftrightarrow |1-3k|=\sqrt{5}\sqrt{k^2+1} \quad |()^2$ mol. puol. ≥0

$\Leftrightarrow (1-3k)^2=(\sqrt{5}\cdot\sqrt{k^2+1})^2$

$\Leftrightarrow 1-6k+9k^2=5(k^2+1) \Leftrightarrow 1-6k+9k^2=5k^2+5$

$\Leftrightarrow 4k^2-6k-4=0 \Leftrightarrow k=\begin{cases} -\frac{1}{2} \\ 2 \end{cases}$ (rats. kaava)

$k=-\frac{1}{2}$: $-\frac{1}{2}x-y-4\cdot(-\frac{1}{2})-1=0 \Leftrightarrow y=-\frac{1}{2}x+1$

$k=2$: $2x-y-4\cdot2-1=0 \Leftrightarrow y=2x-9$

13.5

$x^2+y^2-4x-6=0$

$\Leftrightarrow x^2-4x+y^2=6 \quad |+2^2$

$\Leftrightarrow (x^2-2\cdot x\cdot2+2^2)+y^2=6+2^2$

$\Leftrightarrow (x-2)^2+y^2=10$, kp $=(2,0)$, $r=\sqrt{10}$

tangentti: $y=-3x+b \Leftrightarrow 3x+y-b=0$

kp:n etäisyys tangentista = säde:

$$d=\frac{|3\cdot2+0-b|}{\sqrt{3^2+1^2}}=\frac{|6-b|}{\sqrt{10}}=\sqrt{10} \quad |\cdot\sqrt{10}$$

$\Leftrightarrow |6-b|=10 \Leftrightarrow 6-b=\pm10 \Leftrightarrow b=6\pm10=\begin{cases} 16 \\ -4 \end{cases}$

$\Rightarrow$ tangentit: $y=-3x-4$ tai $y=-3x+16$

13.7

$(x-1)^2+(y-5)^2=34$, kp $=A=(1,5)$, $r=\sqrt{34}$

a) $B=(-1,0)$: $|AB|=\sqrt{(-1-1)^2+(0-5)^2}=\sqrt{29}<r$

$\Rightarrow$ B on ympyrän sisällä $\Rightarrow$ 0 tangenttia

b) $C=(6,2)$: $|AC|=\sqrt{(6-1)^2+(2-5)^2}=\sqrt{34}=r$

$\Rightarrow$ C on ympyrällä $\Rightarrow$ 1 tangentti

c) $D=(5,10)$: $|AD|=\sqrt{(5-1)^2+(10-5)^2}=\sqrt{41}>r$

$\Rightarrow$ D on ympyrän ulkopuolella $\Rightarrow$ 2 tangenttia

13.10

$(x+1)^2+(y-6)^2=100$, kp $=A=(-1,6)$, $r=\sqrt{100}=10$

$B=(5,14)$: $(5+1)^2+(14-6)^2=100 \Leftrightarrow 36+64=100$ ✓

$\Rightarrow$ B on ympyrällä; $k_{AB}=\frac{\Delta y}{\Delta x}=\frac{14-6}{5-(-1)}=\frac{8}{6}=\frac{4}{3}$

$k_t\cdot k_{AB}=-1 \Leftrightarrow k_t=-\frac{1}{k_{AB}}=-\frac{1}{\frac{4}{3}}=-\frac{3}{4}$

tangentti: $y-14=-\frac{3}{4}(x-5) \quad |\cdot4 \Leftrightarrow 4y-56=-3(x-5)$

$\Leftrightarrow 3x+4y-71=0$