y:in etäisyys suorasta: $d = \frac{|4\cdot 2 - 3\cdot 1 + 20|}{\sqrt{4^2+(-3)^2}} = \frac{125}{5} = \frac{25}{5} = 5$

$\Rightarrow$ pienin etäisyys suorasta: $d - r = 5 - 2 = \underline{3}$

10.11 $(x+1)^2 + (y-50)^2 = 22+1$

a) $r = 7 \Rightarrow r^2 = 22+1 = 49 \Leftrightarrow 22 = 48 \Leftrightarrow \underline{a = 24}$

b) $r = 0 \Rightarrow r^2 = 22+1 = 0 \Leftrightarrow \underline{a = -\frac{1}{2}}$

$\Rightarrow \underbrace{(x+1)^2}_{\geq 0} + \underbrace{(y-50)^2}_{\geq 0} = 0 \Leftrightarrow x = -1$ ja $y = 50$

$\Rightarrow$ piste $\underline{(-1, 50)}$

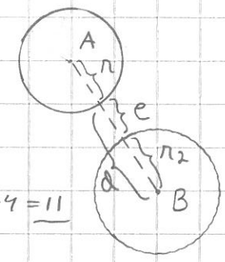
10.12 a) $\underline{k_p} = (0,1)$, $r = 4$: $x^2 + (y-1)^2 = 16$

b) $A = (-4,-1)$, $|\underline{k_p} A| = \sqrt{(-4-0)^2 + (-1-1)^2} = \sqrt{16+4} = \sqrt{20} > r$

$\Rightarrow$ A on ympyrän ulkopuolella

10.17 $(x-1)^2 + (y-5)^2 = 16$

$\Rightarrow \underline{k_p} = A = (1,5)$, $r = \sqrt{16} = 4$



a) $B = (13,-4)$

$|AB| = \sqrt{(13-1)^2 + (-4-5)^2} = 15 > r$

$\Rightarrow$ lyhin etäisyys: $d = |AB| - r = 15 - 4 = \underline{11}$

b) $(x-13)^2 + (y+4)^2 = 36$

$\Rightarrow \underline{k_p} = B = (13,-4)$, $r_2 = \sqrt{36} = 6$

$\Rightarrow$ lyhin etäisyys: $e = |AB| - r - r_2 = 15 - 4 - 6 = \underline{5}$

11.1 $x^2 + y^2 + ax + by + c = 0$

$(0,3)$: $0^2 + 3^2 + a\cdot 0 + b\cdot 3 + c = 0$

$(10,-7)$: $10^2 + (-7)^2 + a\cdot 10 + b\cdot(-7) + c = 0$

$(8,-9)$: $8^2 + (-9)^2 + a\cdot 8 + b\cdot(-9) + c = 0$

$\Leftrightarrow \begin{cases} 3b + c = -9 & (1) \\ 10a - 7b + c = -149 & (2) \\ 8a - 9b + c = -145 & (3) \end{cases}$

(1)-(2): $-10a + 10b = 140 \quad |\cdot(-4)$

(1)-(3): $-8a + 12b = 136 \quad |\cdot 5$

$20b = 120 \Leftrightarrow b = 6$

$-10a + 10\cdot 6 = 140 \Leftrightarrow 10a = -80 \Leftrightarrow a = -8$

(1): $c = -9 - 3\cdot 6 = -27$

$\Rightarrow \underline{x^2 + y^2 - 8x + 6y - 27 = 0}$

11.7 $x^2 + y^2 + 50x - 2y + 606 = 0$

$\Leftrightarrow x^2 + 50x + y^2 - 2y = -606 \quad |+25^2 + 1^2$

$\Leftrightarrow (x^2 + 2\cdot x\cdot 25 + 25^2) + (y^2 - 2\cdot y\cdot 1 + 1^2) = -606 + 25^2 + 1^2$

$\Leftrightarrow (x+25)^2 + (y-1)^2 = 20$, $r = \sqrt{20}$

a) piiri: $2\pi r = 2\pi\cdot\sqrt{20} = 2\pi\sqrt{4\cdot 5} = \underline{4\sqrt{5}\pi}$

b) pinta-ala: $\pi r^2 = \pi(\sqrt{20})^2 = \underline{20\pi}$

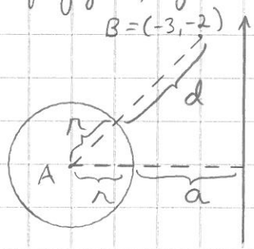
11.10 $x^2 + y^2 + 16x + 14y + 95 = 0$

$\Leftrightarrow x^2 + 16x + y^2 + 14y = -95 \quad |+8^2 + 7^2$

$\Leftrightarrow (x^2 + 2\cdot x\cdot 8 + 8^2) + (y^2 + 2\cdot y\cdot 7 + 7^2) = -95 + 8^2 + 7^2$

$\Leftrightarrow (x+8)^2 + (y+7)^2 = 18$

ympyrä, $\underline{k_p} = A = (-8,-7)$, $r = \sqrt{18} = \sqrt{9\cdot 2} = 3\sqrt{2}$



a) $|AB| = \sqrt{(-8-(-3))^2 + (-7-(-2))^2}$

$= \sqrt{25+25} = \sqrt{50} = \sqrt{25\cdot 2} = 5\sqrt{2}$

$\Rightarrow d = |AB| - r = 5\sqrt{2} - 3\sqrt{2} = \underline{2\sqrt{2}}$

b) $a = |-8| - r = \underline{8 - 3\sqrt{2}}$

11.13 a) $x^2 + y^2 - 4x + 6y + 9 = 0$

$\Leftrightarrow x^2 - 4x + y^2 + 6y = -9 \quad |+2^2 + 3^2$

$\Leftrightarrow (x^2 - 2\cdot x\cdot 2 + 2^2) + (y^2 + 2\cdot y\cdot 3 + 3^2) = -9 + 2^2 + 3^2$

$\Leftrightarrow (x-2)^2 + (y+3)^2 = 4 \Rightarrow \underline{k_p} = (2,-3)$, $r = \sqrt{4} = 2$

b) $x^2 + y^2 + 4x - 6y + 13 = 0$

$\Leftrightarrow x^2 + 4x + y^2 - 6y = -13 \quad |+2^2 + 3^2$

$\Leftrightarrow (x^2 + 2\cdot x\cdot 2 + 2^2) + (y^2 - 2\cdot y\cdot 3 + 3^2) = -13 + 2^2 + 3^2$

$\Leftrightarrow (x+2)^2 + (y-3)^2 = 0 \Rightarrow$ piste $\underline{(-2, 3)}$

c) $4x^2 + 4y^2 + 4x - 7 = 0 \quad |:4$

$\Leftrightarrow x^2 + y^2 + x - \frac{7}{4} = 0$

$\Leftrightarrow x^2 + x + y^2 = \frac{7}{4} \quad |+(\frac{1}{2})^2$

$\Leftrightarrow (x^2 + 2\cdot x\cdot\frac{1}{2} + (\frac{1}{2})^2) + y^2 = \frac{7}{4} + (\frac{1}{2})^2$

$\Leftrightarrow (x + \frac{1}{2})^2 + y^2 = 2$

ympyrä, $\underline{k_p} = (-\frac{1}{2}, 0)$, $r = \sqrt{2}$

11.15 b) $x^2 + y^2 + 2px - 4py + 4p^2 = 0$

$\Leftrightarrow x^2 + 2px + y^2 - 4py = -4p^2 \quad |+p^2 + (2p)^2$

$\Leftrightarrow (x^2 + 2\cdot x\cdot p + p^2) + (y^2 - 2\cdot y\cdot 2p + (2p)^2) = -4p^2 + p^2 + (2p)^2$

$\Leftrightarrow (x+p)^2 + (y-2p)^2 = p^2$

esittää ympyrää $\Leftrightarrow p^2 > 0 \Leftrightarrow \underline{p \neq 0}$, jolloin

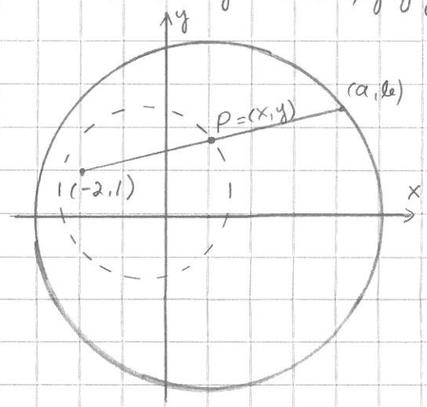
$\underline{k_p} = (-p, 2p)$ ja $r = \sqrt{p^2} = \underline{|p|}$

11.16 $x^2 + y^2 - 2x - 15 = 0$

$\Leftrightarrow x^2 - 2x + y^2 = 15 \quad |+1^2$

$\Leftrightarrow (x^2 - 2\cdot x\cdot 1 + 1^2) + y^2 = 15 + 1^2$

$\Leftrightarrow (x-1)^2 + y^2 = 16$, ympyrä $\underline{k_p} = (1,0)$, $r = \sqrt{16} = 4$



1° Olkoon $P = (x,y)$ mielivaltainen säätön piste (jonain reaalipiste)

2° yhteys x:lle ja y:lle

$P = (x,y) = (\frac{a-2}{2}, \frac{b+1}{2})$

$\Leftrightarrow \begin{cases} x = \frac{a-2}{2} & \Leftrightarrow a = 2x+2 \\ y = \frac{b+1}{2} & \Leftrightarrow b = 2y-1 \end{cases}$

$(a,b) = (2x+2, 2y-1)$ on annetulla ympyrällä:

$(2x+2)^2 + (2y-1)^2 - 2(2x+2) - 15 = 0$

$\Leftrightarrow (4x^2 + 8x + 4) + (4y^2 - 4y + 1) - 4x - 4 - 15 = 0$

$\Leftrightarrow 4x^2 + 4y^2 + 4x - 4y - 14 = 0 \quad |:4$

$\Leftrightarrow x^2 + y^2 + x - y - \frac{7}{2} = 0$

$\Leftrightarrow x^2 + x + y^2 - y = \frac{7}{2} \quad |+(\frac{1}{2})^2 + (\frac{1}{2})^2$

$\Leftrightarrow (x^2 + 2\cdot x\cdot\frac{1}{2} + (\frac{1}{2})^2) + (y^2 - 2\cdot y\cdot\frac{1}{2} + (\frac{1}{2})^2) = \frac{7}{2} + (\frac{1}{2})^2 + (\frac{1}{2})^2$

$\Leftrightarrow (x + \frac{1}{2})^2 + (y - \frac{1}{2})^2 = 4$

ympyrä, $\underline{k_p} = (-\frac{1}{2}, \frac{1}{2})$, $r = \sqrt{4} = 2$

11.21 $x^2 + y^2 - 18x + 2y - 78 = 0$

$\Leftrightarrow x^2 - 18x + y^2 + 2y = 78 \quad |+9^2 + 1^2$

$\Leftrightarrow (x^2 - 2\cdot x\cdot 9 + 9^2) + (y^2 + 2\cdot y\cdot 1 + 1^2) = 78 + 9^2 + 1^2$

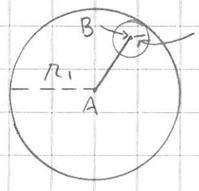
$\Leftrightarrow (x-9)^2 + (y+1)^2 = 160$, $\underline{k_p} = A = (9,-1)$, $r_1 = \sqrt{160} = 4\sqrt{10}$

$x^2 + y^2 - 4y - 6 = 0$

$\Leftrightarrow x^2 + y^2 - 4y = 6 \quad |+2^2$

$\Leftrightarrow x^2 + (y^2 - 2\cdot y\cdot 2 + 2^2) = 6 + 2^2$

$\Leftrightarrow x^2 + (y-2)^2 = 10$, $\underline{k_p} = B = (0,2)$, $r_2 = \sqrt{10}$



$|AB| = \sqrt{(9-0)^2 + (-1-2)^2} = \sqrt{90} = \sqrt{9\cdot 10} = 3\sqrt{10}$

$|AB| + r_2 = 3\sqrt{10} + \sqrt{10} = 4\sqrt{10} = r_1$

$\Rightarrow$ ympyrät sivuavat toisiaan sisäpuolisesti

12.1 $\begin{cases} y = 5x \\ x^2 + y^2 = 104 \end{cases}$ sij. $\Rightarrow x^2 + (5x)^2 = 104 \Leftrightarrow x^2 + 25x^2 = 104$

$\Leftrightarrow 26x^2 = 104 \quad |:26 \Leftrightarrow x^2 = 4 \quad |\sqrt{} \Leftrightarrow x = \pm 2$

$x = 2$: $y = 10$; $x = -2$: $y = -10$ $\Rightarrow \underline{(-2,-10)}$ ja $\underline{(2,10)}$