

5. $x^2 + y^2 + 2x - 6y - 3 = 0$
 $(\Rightarrow) x^2 + 2x + y^2 - 6y = 3 \quad | +1^2 + 3^2$
 $(\Rightarrow) (x^2 + 2 \cdot x \cdot 1 + 1^2) + (y^2 - 2 \cdot y \cdot 3 + 3^2) = 3 + 1^2 + 3^2$
 $(\Rightarrow) (x+1)^2 + (y-3)^2 = 13$

ympyrä, $S_p = A = (-1, 3), r = \sqrt{13}$

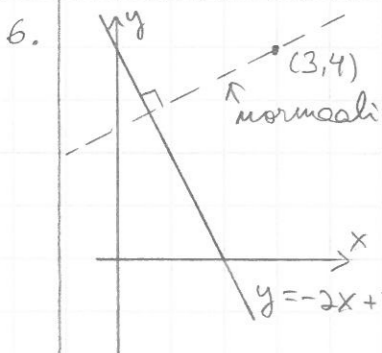
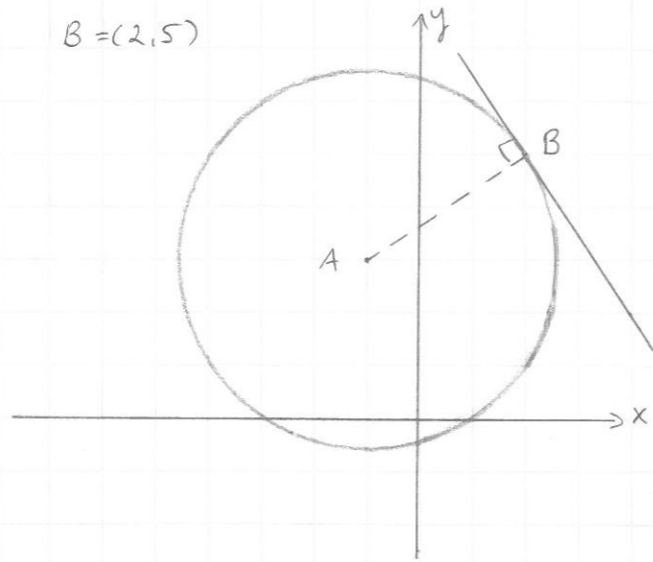
$B = (2, 5): 2^2 + 5^2 + 2 \cdot 2 - 6 \cdot 5 - 3 = 0 \Rightarrow 0 = 0 \%$

$\Rightarrow B$ on ympyrällä

$k_{AB} = \frac{\Delta y}{\Delta x} = \frac{5-3}{2-(-1)} = \frac{2}{3}$

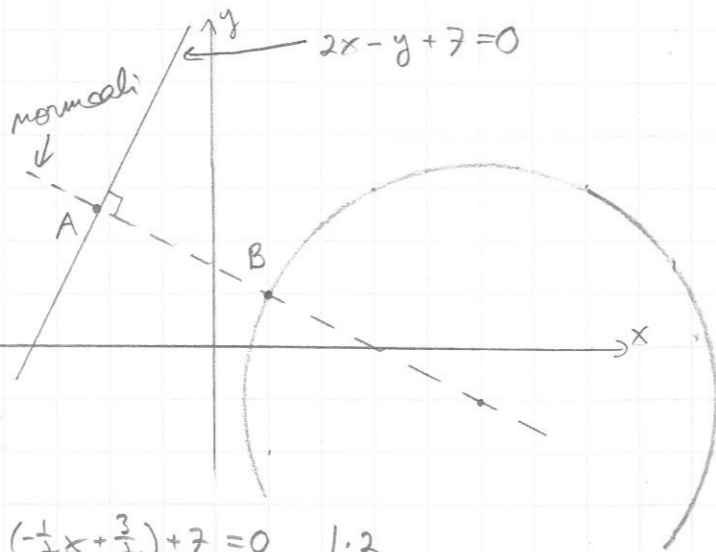
$k_t \cdot k_{AB} = -1 \Rightarrow k_t = -\frac{1}{k_{AB}} = -\frac{1}{\frac{2}{3}} = -\frac{3}{2}$

tangentti: $y - 5 = -\frac{3}{2}(x - 2) \Rightarrow y = -\frac{3}{2}x + 8$



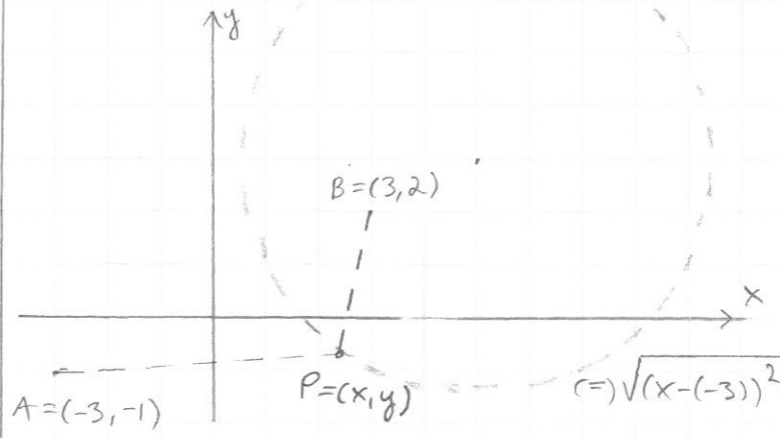
$y = -2x + 4 \Rightarrow k_1 = -2$
 $k_1 \cdot k_2 = -1 \Rightarrow k_2 = -\frac{1}{k_1} = -\frac{1}{-2} = \frac{1}{2}$
 normaali: $y - 4 = \frac{1}{2}(x - 3) \Rightarrow y = \frac{1}{2}x + \frac{5}{2}$
 leikkauspiste: $\begin{cases} y = \frac{1}{2}x + \frac{5}{2} \\ y = -2x + 4 \end{cases} \rightarrow$
 $\Rightarrow \frac{1}{2}x + \frac{5}{2} = -2x + 4 \quad | \cdot 2 \Rightarrow x + 5 = -4x + 8$
 $\Rightarrow 5x = 3 \Rightarrow x = \frac{3}{5} \Rightarrow y = -2 \cdot \frac{3}{5} + 4 = \frac{14}{5} \Rightarrow (\frac{3}{5}, \frac{14}{5})$

7. $x^2 + y^2 - 10x + 2y + 6 = 0$
 $(\Rightarrow) x^2 - 10x + y^2 + 2y = -6 \quad | +5^2 + 1^2$
 $(\Rightarrow) (x^2 - 2 \cdot x \cdot 5 + 5^2) + (y^2 + 2 \cdot y \cdot 1 + 1^2) = -6 + 5^2 + 1^2$
 $(\Rightarrow) (x-5)^2 + (y+1)^2 = 20$
 ympyrä, $S_p = (5, -1), r = \sqrt{20}$
 $2x - y + 7 = 0 \Rightarrow y = 2x + 7 \Rightarrow k_1 = 2$
 $k_1 \cdot k_2 = -1 \Rightarrow k_2 = -\frac{1}{k_1} = -\frac{1}{2}$
 normaali: $y - (-1) = -\frac{1}{2}(x - 5) \Rightarrow y = -\frac{1}{2}x + \frac{3}{2}$



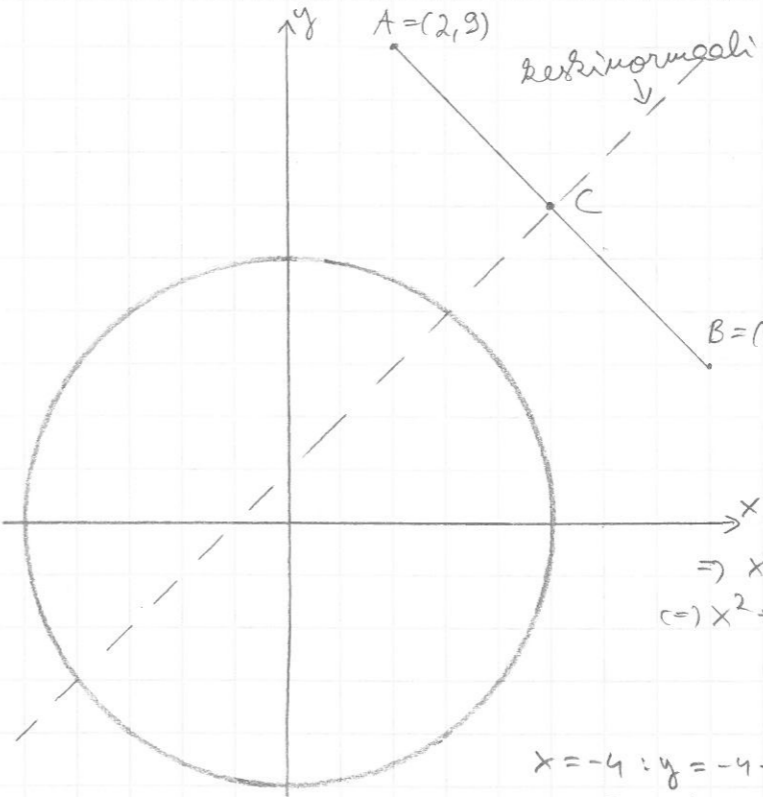
leikkauspiste: $\begin{cases} y = -\frac{1}{2}x + \frac{3}{2} \\ 2x - y + 7 = 0 \end{cases} \rightarrow$
 $\Rightarrow 2x - (-\frac{1}{2}x + \frac{3}{2}) + 7 = 0 \quad | \cdot 2$
 $\Rightarrow 4x + x - 3 + 14 = 0 \Rightarrow 5x = -11 \Rightarrow x = -\frac{11}{5} \Rightarrow y = 2 \cdot (-\frac{11}{5}) + 7 = \frac{13}{5} \Rightarrow A = (-\frac{11}{5}, \frac{13}{5})$
 leikkauspiste: $\begin{cases} y = -\frac{1}{2}x + \frac{3}{2} \\ x^2 + y^2 - 10x + 2y + 6 = 0 \end{cases} \rightarrow$
 $\Rightarrow x^2 + (-\frac{1}{2}x + \frac{3}{2})^2 - 10x + 2(-\frac{1}{2}x + \frac{3}{2}) + 6 = 0$
 $\Rightarrow x^2 + \frac{1}{4}x^2 - \frac{3}{2}x + \frac{9}{4} - 10x - x + 3 + 6 = 0 \quad | \cdot 4$
 $\Rightarrow 5x^2 - 50x + 45 = 0 \Rightarrow x = \begin{cases} 1 \in \text{vasen puoli} \\ 9 \in \text{oikea puoli} \end{cases}$
 $y = -\frac{1}{2} \cdot 1 + \frac{3}{2} = 1 \Rightarrow B = (1, 1)$

8. Käyrän yhtälön määrittely:
 1° Oletaan $P = (x, y)$ mielivaltaisen käyrän piste
 2° yhtälö x :lle ja y :lle
 $|AP| = 2|BP|$
 $\text{mol. puol.} \geq 0$



$(\Rightarrow) \sqrt{(x-(-3))^2 + (y-(-1))^2} = 2\sqrt{(x-3)^2 + (y-2)^2} \quad | (\cdot)^2$
 3° Siirrenyt: $(\Rightarrow) (x+3)^2 + (y+1)^2 = 4[(x-3)^2 + (y-2)^2]$
 $(\Rightarrow) x^2 + 6x + 9 + y^2 + 2y + 1 = 4[x^2 - 6x + 9 + y^2 - 4y + 4]$
 $(\Rightarrow) -3x^2 - 3y^2 + 30x + 18y - 42 = 0 \quad | : (-3)$
 $(\Rightarrow) x^2 + y^2 - 10x - 6y + 14 = 0$
 $(\Rightarrow) x^2 - 10x + y^2 - 6y = -14 \quad | +5^2 + 3^2$
 $(\Rightarrow) (x^2 - 2 \cdot x \cdot 5 + 5^2) + (y^2 - 2 \cdot y \cdot 3 + 3^2) = -14 + 5^2 + 3^2$
 $(\Rightarrow) (x-5)^2 + (y-3)^2 = 20$, ympyrä, $S_p = (5, 3), r = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

8'. $A = (2, 9), B = (8, 3)$
 keskinormaali: $k_{AB} = \frac{\Delta y}{\Delta x} = \frac{3-9}{8-2} = \frac{-6}{6} = -1$
 keskipiste: $C = (\frac{2+8}{2}, \frac{9+3}{2}) = (5, 6)$
 keskinormaali:
 $k \cdot k_{AB} = -1 \Rightarrow k = -\frac{1}{k_{AB}} = -\frac{1}{-1} = 1$
 $y - 6 = 1 \cdot (x - 5) \Rightarrow y = x + 1$
 leikkauspiste: $\begin{cases} y = x + 1 \\ x^2 + y^2 = 5^2 \end{cases} \rightarrow$
 $\Rightarrow x^2 + (x+1)^2 = 25$
 $(\Rightarrow) x^2 + x^2 + 2x + 1 = 25 \Rightarrow 2x^2 + 2x - 24 = 0$
 $(\Rightarrow) x = \begin{cases} -4 \\ 3 \end{cases}$
 $x = -4: y = -4 + 1 = -3$
 $x = 3: y = 3 + 1 = 4$
 vast. $(-4, -3)$ tai $(3, 4)$



9. alaspäin aukeava paraabeli: $y = ax^2 + bx + c$
 $(10, 0): c = 0 \quad (1)$
 $(72, 0): a \cdot 72^2 + b \cdot 72 + c = 0 \quad (2)$
 $(48, 13,5): a \cdot 48^2 + b \cdot 48 + c = 13,5 \quad (3)$
 $-48 \cdot (2) + 72 \cdot (3): -82944a = 972 \Rightarrow a = -\frac{3}{256}$
 $(2): b = -\frac{1}{72} \cdot (-\frac{3}{256} \cdot 72^2) = \frac{27}{32} \Rightarrow y = -\frac{3}{256}x^2 + \frac{27}{32}$
 Korkeim kohta kun $x = \frac{0+72}{2} = 36$ (symmetria)
 $h = -\frac{3}{256} \cdot 36^2 + \frac{27}{32} = \frac{243}{16} = 15,1875 \approx 15,2 \text{ (m)}$

