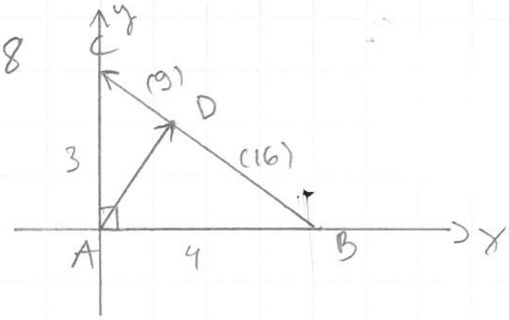
21.18



$\bar{BC} = -4\bar{i} + 3\bar{j}$

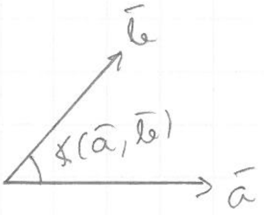
$\bar{AD} = \bar{AC} + \bar{CD} = 3\bar{j} + \frac{9}{25}\bar{CB} = 3\bar{j} + \frac{9}{25}(-3\bar{j} + 4\bar{i})$

$= 3\bar{j} - \frac{27}{25}\bar{j} + \frac{36}{25}\bar{i} = \frac{36}{25}\bar{i} + \frac{48}{25}\bar{j}$

$\bar{BC} \cdot \bar{AD} = (-4) \cdot \frac{36}{25} + 3 \cdot \frac{48}{25} = 0$

$\Leftrightarrow \bar{BC} \perp \bar{AD}$

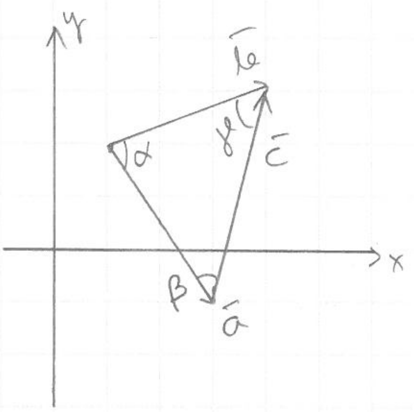
22. Vektorien välinen kulma



$$\boxed{\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos(\bar{a}, \bar{b})}$$

$$\Leftrightarrow \cos(\bar{a}, \bar{b}) = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|}$$

22.13



$\bar{a} = 2\bar{i} - 3\bar{j} \Rightarrow |\bar{a}| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

$\bar{b} = 3\bar{i} + \bar{j} \Rightarrow |\bar{b}| = \sqrt{3^2 + 1^2} = \sqrt{10}$

$\cos(\bar{a}, \bar{b}) = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|} = \frac{2 \cdot 3 + (-3) \cdot 1}{\sqrt{13} \cdot \sqrt{10}} = \frac{3}{\sqrt{13}\sqrt{10}}$

$\Rightarrow \alpha = ∢(\bar{a}, \bar{b}) \approx 74{,}74° \approx \underline{75°}$

$-\bar{a} = -2\bar{i} + 3\bar{j} \Rightarrow |-a| = |\bar{a}| = \sqrt{13}$

$\bar{c} = -\bar{a} + \bar{b} = \bar{i} + 4\bar{j} \Rightarrow |\bar{c}| = \sqrt{1^2 + 4^2} = \sqrt{17}$

$\cos(-\bar{a}, \bar{c}) = \frac{-\bar{a} \cdot \bar{c}}{|-\bar{a}||\bar{c}|} = \frac{-2 \cdot 1 + 3 \cdot 4}{\sqrt{13}\ \sqrt{17}} = \frac{10}{\sqrt{13}\sqrt{17}} \Rightarrow \beta = ∢(-\bar{a}, \bar{c}) \approx 47{,}73°$

$\approx \underline{48°}$

$-\bar{b} = -3\bar{i} - \bar{j}$

$-\bar{c} = -\bar{i} - 4\bar{j}$

$\cos(-\bar{b}, -\bar{c}) = \frac{-\bar{b} \cdot (-c)}{|-\bar{b}||-\bar{c}|} = \frac{-3 \cdot (-1) + (-1) \cdot (-4)}{\sqrt{10}\ \sqrt{17}} = \frac{7}{\sqrt{10}\sqrt{17}}$

$\Rightarrow \gamma = ∢(-\bar{b}, -\bar{c}) \approx 57{,}53°$

$\approx \underline{58°}$

Tark. $\alpha + \beta + \gamma = 181°$ (↯)

pyöristykset