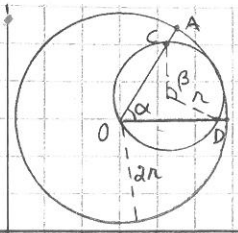
11.16

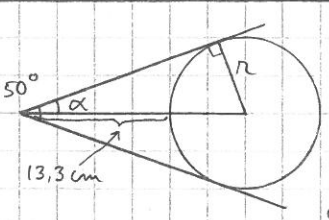


$\beta = 2\alpha$ (kehäkulmalause)

$\underline{CD} = \frac{\beta}{360°} \cdot 2\pi r = \frac{2\alpha}{360°} \cdot 2\pi r$

$= \frac{\alpha}{360°} \cdot 2\pi \cdot 2r = \underline{AB}$ ⇒ väite

12.2



$\alpha = \frac{50°}{2} = 25°$ (symmetria)

$\sin 25° = \frac{r}{13{,}3 + r} \quad | \cdot (13{,}3 + r)$

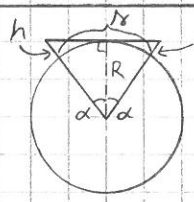
$\Leftrightarrow \sin 25° \cdot (13{,}3 + r) = r$

$\Leftrightarrow 13{,}3 \sin 25° + r \sin 25° = r$

$\Leftrightarrow 13{,}3 \sin 25° = r - r \sin 25° = r(1 - \sin 25°) \quad | : (1 - \sin 25°)$

$\Leftrightarrow r = \frac{13{,}3 \sin 25°}{1 - \sin 25°} \approx 9{,}73502 \approx \underline{9{,}7\ (cm)}$

12.4



$r = 110$ km, $R = 6370$ km

$r = \frac{2\alpha}{360°} \cdot 2\pi R \quad | \cdot \frac{360°}{4\pi R}$

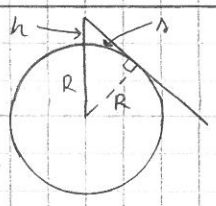
$\Leftrightarrow \alpha = \frac{110\text{ km} \cdot 360°}{4\pi \cdot 6370\text{ km}} \approx 0{,}494705°$

$\cos\alpha = \frac{R}{R+h} \quad | \cdot \frac{R+h}{\cos\alpha} \quad \Leftrightarrow \quad R + h = \frac{R}{\cos\alpha}$

$\Leftrightarrow h = \frac{R}{\cos\alpha} - R = \frac{6370\text{ km}}{\cos 0{,}494705°} - 6370\text{ km} \approx 0{,}23745\text{ km}$

⇒ vähintään $\underline{240\text{ m}}$

12.5



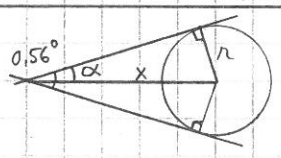
$h = 3718$ m, $R = 6370$ km

$\cos\alpha = \frac{R}{R+h} = \frac{6370\text{ km}}{6370\text{ km} + 3{,}718\text{ km}}$

$\Rightarrow \alpha \approx 1{,}957°$

$r = \frac{\alpha}{360°} \cdot 2\pi R = \frac{1{,}957°}{360°} \cdot 2\pi \cdot 6370\text{ km} \approx 217{,}587\text{ km} \approx \underline{218\text{ km}}$

12.7



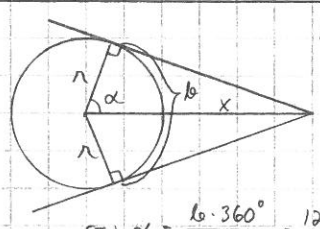
$r = 1737$ km

$\alpha = \frac{0{,}56°}{2} = 0{,}28°$ (symmetria)

$\sin\alpha = \frac{r}{x} \quad | \cdot \frac{x}{\sin\alpha}$

$\Leftrightarrow x = \frac{r}{\sin\alpha} = \frac{1737\text{ km}}{\sin 0{,}28°} \approx 355\,439{,}9\text{ km} \approx \underline{355\,400\text{ km}}$

12.12



$b = 12{,}3$ cm

$2\pi r = 26{,}4$ cm $\quad | : 2\pi$

$\Leftrightarrow r = \frac{26{,}4\text{ cm}}{2\pi} \approx 4{,}202$ cm

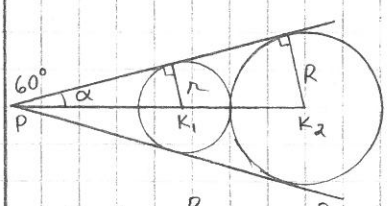
$b = \frac{2\alpha}{360°} \cdot 2\pi r \quad | \cdot \frac{360°}{4\pi r}$

$\Leftrightarrow \alpha = \frac{b \cdot 360°}{4\pi r} = \frac{12{,}3\text{ cm} \cdot 360°}{4\pi \cdot 4{,}202\text{ cm}} \approx 83{,}8636°$

$\cos\alpha = \frac{r}{x} \quad | \cdot \frac{x}{\cos\alpha} \quad \Leftrightarrow x = \frac{r}{\cos\alpha} = \frac{4{,}202\text{ cm}}{\cos 83{,}8636°} \approx 39{,}3067$ cm

⇒ etäisyys pullosta vähintään: $x - r \approx 35{,}105$ cm $\approx \underline{35{,}1\text{ cm}}$

12.15



$PK_1 = 10$

$\alpha = \frac{60°}{2} = 30°$ (symmetria)

$\sin 30° = \frac{r}{10} \quad | \cdot 10$

$\Leftrightarrow r = 10 \sin 30° = 10 \cdot \frac{1}{2} = 5$

$\sin 30° = \frac{R}{10 + r + R} = \frac{R}{10 + 5 + R} = \frac{1}{2} \quad | \cdot 2$ ⇒ $2R = 15 + R$ ⇔ $\underline{R = 15}$

13.3

$V = 0{,}0900\text{ m} \cdot 0{,}0400\text{ m} \cdot 0{,}0076\text{ m} = 2{,}736 \cdot 10^{-5}\text{ m}^3$

tiheys: $\rho = \frac{m}{V} = \frac{0{,}5000\text{ kg}}{2{,}736 \cdot 10^{-5}\text{ m}^3} \approx 18\,274{,}9\ \frac{\text{kg}}{\text{m}^3} < 19\,320\ \frac{\text{kg}}{\text{m}^3}$

⇒ ei ole puhdasta kultaa

13.4

ulkomitat: $a = 80$ cm, $b = 60$ cm, $c = 40$ cm

seinämien paksuus: $d = 8{,}0$ cm

sisämitat: $a - 2d = 64$ cm, $b - 2d = 44$ cm, $c - 2d = 24$ cm

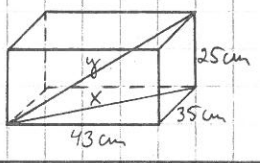
a) tilavuus: $V = 6{,}4\text{ dm} \cdot 4{,}4\text{ dm} \cdot 2{,}4\text{ dm} = 67{,}584\text{ dm}^3 \approx \underline{68}$

b) styrox-seinämien tilavuus:

$V_s = abc - V = 8{,}0\text{ dm} \cdot 6{,}0\text{ dm} \cdot 4{,}0\text{ dm} - 67{,}584\text{ dm}^3 = 124{,}416$

$m = \rho V = 21\ \frac{\text{g}}{\text{dm}^3} \cdot 124{,}416\text{ dm}^3 = 2612{,}736\text{ g} \approx \underline{2{,}6\text{ kg}}$

13.7



a) Pythagoras: $x^2 = 43^2 + 35^2 \quad |$

$\Leftrightarrow x \approx 55{,}444 \approx \underline{55\ (cm)}$

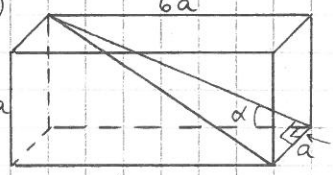
b) Pythagoras: $y^2 = 43^2 + 35^2 + 25^2$

$\Leftrightarrow y \approx 60{,}819 \approx \underline{60\ (cm)}$

pyöristys alaspäin

13.9

a)

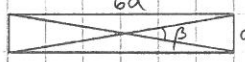


$\tan\alpha = \frac{2a}{6a} = \frac{1}{3}$

$\Rightarrow \alpha \approx 18{,}435° \approx \underline{18{,}4°}$

tasojen leikkaussuora

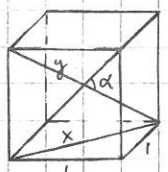
b) ylhäältä:



$\tan\frac{\beta}{2} = \frac{\frac{1}{2}a}{\frac{1}{2} \cdot 6a} = \frac{1}{6}$

$\Leftrightarrow \frac{\beta}{2} \approx 9{,}462° \Leftrightarrow \beta \approx 18{,}92° \approx \underline{18,}$

13.12



a) Pythagoras: $x^2 = 1^2 + 1^2 = 2 \Leftrightarrow \underline{x = \sqrt{2}}$

b) $\quad\quad : y^2 = 1^2 + 1^2 + 1^2 = 3 \Leftrightarrow \underline{y = \sqrt{3}}$

c) kosinilause:

$1^2 = \left(\frac{y}{2}\right)^2 + \left(\frac{y}{2}\right)^2 - 2 \cdot \frac{y}{2} \cdot \frac{y}{2} \cdot \cos\alpha$

$\Leftrightarrow 1 = \frac{3}{4} + \frac{3}{4} - \frac{3}{2} \cdot \cos\alpha$

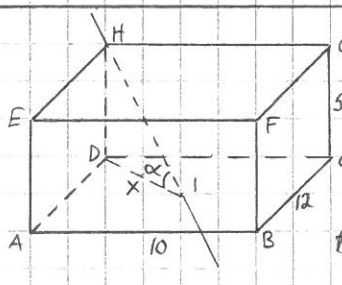
$\Leftrightarrow \cos\alpha = \frac{1}{3} \Rightarrow \alpha \approx 70{,}5288° \approx \underline{71°}$

TAI: $\sin\frac{\alpha}{2} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \Rightarrow \frac{\alpha}{2} \approx 35{,}2644° \Leftrightarrow \alpha \approx 70{,}528 \approx 71°$

TAI: $\cos\frac{\alpha}{2} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \frac{\alpha}{2} \approx 35{,}2644°$

TAI: $\tan\frac{\alpha}{2} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\alpha}{2} \approx 35{,}2644°$ ⌟

13.17



$x = \frac{1}{2}\sqrt{10^2 + 12^2}$

$= \frac{1}{2}\sqrt{244} = \frac{1}{2}\sqrt{4 \cdot 61}$

$= \frac{1}{2} \cdot 2\sqrt{61} = \sqrt{61}$

(puolet pohjataltaan kuutiosta)

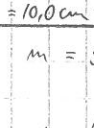
$\tan\alpha = \frac{5}{\sqrt{61}} \Rightarrow \alpha \approx 32{,}63° \approx \underline{33°}$

14.2

a) $V = 1{,}25\text{ dm} \cdot 1{,}00\text{ dm} \cdot 0{,}145\text{ dm} = 0{,}18125\text{ dm}^3 \approx \underline{0{,}181\text{ l}}$

b) $V = \pi \cdot (15\text{ dm})^2 \cdot 40\text{ dm} \approx 28\,274{,}3\text{ dm}^3 \approx \underline{28\,000\text{ l}}$

14.5



$h = 10{,}0$ cm $= 1{,}00$ dm

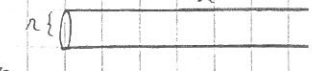
$V = \pi r^2 h = 1\ (\text{dm}^3) \Leftrightarrow r = \sqrt{\frac{1}{\pi}} \approx 0{,}56419\ (\text{dm})$

$A = \pi r^2 + 2\pi r \cdot h = \pi \cdot (0{,}56419\text{ dm})^2 + 2\pi \cdot 0{,}5641 \cdot 1{,}00\text{ dm}$

$\approx 4{,}54491\text{ dm}^2 \approx \underline{454\text{ cm}^2}$

14.8

$m = \rho V = \rho \cdot \pi r^2 h$



$\Leftrightarrow h = \frac{m}{\rho \pi r^2} = \frac{0{,}36\text{ kg}}{8960\ \frac{\text{kg}}{\text{m}^3} \cdot \pi \cdot \left(\frac{0{,}0012\text{ m}}{2}\right)^2} \approx 35{,}5257\text{ m} \approx \underline{36\text{ m}}$