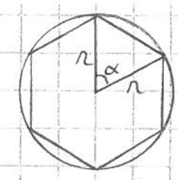
**9.5**

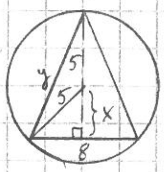



$\alpha = \frac{360°}{6} = 60°$

a) $\frac{A_y}{A_k} = \frac{\pi r^2}{6 \cdot \frac{1}{2} \cdot r \cdot r \cdot \sin 60°} = \frac{\pi}{3 \cdot \frac{\sqrt{3}}{2}} = \frac{2\pi}{3\sqrt{3}}$

b) $\frac{2\pi}{3\sqrt{3}} - 1 \approx 0{,}2092 \approx \underline{21\,\%}$

**9.6**




Pythagoras: $x^2 + 4^2 = 5^2$

$\Leftrightarrow x = \pm\sqrt{5^2 - 4^2} = \sqrt{9} = 3$

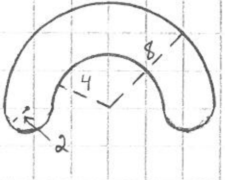
$4^2 + (5+3)^2 = y^2 \quad |\sqrt{\ }$

$\Leftrightarrow y = \pm\sqrt{4^2 + 8^2} = \sqrt{80} = \sqrt{16 \cdot 5} = \underline{4\sqrt{5}}$

**9.8** pyörähdyksiä $n$: $n\pi d = s$

$\Leftrightarrow n = \frac{s}{\pi d} = \frac{200\,m}{\pi \cdot 26 \cdot 0{,}0254\,m} \approx 96{,}3991 \approx \underline{96}$

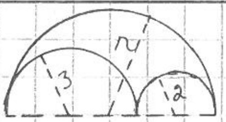
**9.10**




$p = 2 \cdot \frac{1}{2}\pi \cdot 2 \cdot 2 + \frac{1}{2}\pi \cdot 2 \cdot 4 + \frac{1}{2}\pi \cdot 2 \cdot 8 = 16\pi$

$A = \frac{1}{2} \cdot \pi \cdot 8^2 - \frac{1}{2}\pi \cdot 4^2 + \pi \cdot 2^2 = \underline{28\pi}$

**9.16**

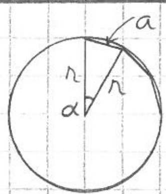



$2r = 2 \cdot 3 + 2 \cdot 2 = 10 \quad \Leftrightarrow r = 5$

piiri: $p = \frac{1}{2} \cdot (2\pi \cdot 5 + 2\pi \cdot 3 + 2\pi \cdot 2) = \underline{10\pi}$

ala: $A = \frac{1}{2}\pi \cdot 5^2 - \frac{1}{2}\pi \cdot 3^2 - \frac{1}{2}\pi \cdot 2^2 = \underline{6\pi}$

**9.18**




$\alpha = \frac{360°}{96} = 3{,}75°$

a) $A_y = \pi r^2 \approx 96 \cdot \frac{1}{2} \cdot r \cdot r \cdot \sin\alpha$

$\Leftrightarrow \pi \approx 48 \sin 3{,}75° \approx 3{,}13935 \approx \underline{3{,}14}$

b) Kosinilause: $a^2 = r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos\alpha$

$\Rightarrow a = \sqrt{2r^2 - 2r^2\cos\alpha} = \sqrt{r^2(2 - 2\cos\alpha)} = r\sqrt{2 - 2\cos\alpha}$

$p_y = \pi \cdot 2r \approx 96a = 96 r\sqrt{2 - 2\cos\alpha} \quad |:2r$

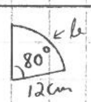
$\Leftrightarrow \pi \approx 48\sqrt{2 - 2\cos 3{,}75°} \approx 3{,}14103 \approx \underline{3{,}14}$

**10.2**


$A_s = \frac{\alpha}{360°}\pi r^2 \quad | \cdot \frac{360°}{\pi r^2}$

$\Leftrightarrow \alpha = \frac{A_s \cdot 360°}{\pi r^2} = \frac{1{,}0\,m^2 \cdot 360°}{\pi \cdot (1{,}5\,m)^2} \approx 50{,}9296° \approx \underline{51°}$

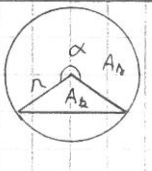
**10.4**




$p = b + 2r = \frac{80°}{360°} \cdot 2\pi \cdot 12\,cm + 2 \cdot 12\,cm \approx 40{,}755\,cm \approx \underline{41\,cm}$

$A_s = \frac{80°}{360°} \cdot \pi(12\,cm)^2 \approx 100{,}531\,cm^2 \approx \underline{100\,cm^2}$

**10.5**



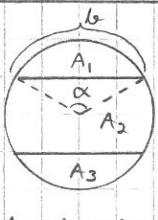

$\alpha = 240° \qquad r = 8{,}0\,cm$

$A_{seg} = A_s + A_k = \frac{\alpha}{360°}\pi r^2 + \frac{1}{2} \cdot r \cdot r \cdot \sin(360° - \alpha)$

$= \frac{240°}{360°} \cdot \pi \cdot (8{,}0\,cm)^2 + \frac{1}{2} \cdot (8{,}0\,cm)^2 \sin 120°$

$\approx 161{,}754\,cm^2 \approx \underline{160\,cm^2}$

**10.10**




$2r = 28\,cm \quad \Leftrightarrow r = 14\,cm$

$b = \frac{\alpha}{360°} \cdot 2\pi r = \frac{1}{3} \cdot 2\pi r \quad | \cdot \frac{360°}{2\pi r}$

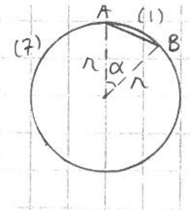
$\Leftrightarrow b = \frac{1}{3} \cdot 360° = 120°$

$A_1 = A_3 = A_s - A_k = \frac{\alpha}{360°} \cdot \pi r^2 - \frac{1}{2} \cdot r \cdot r \cdot \sin\alpha$

$= \frac{120°}{360°}\pi(14\,cm)^2 - \frac{1}{2} \cdot (14\,cm)^2 \cdot \sin 120° \approx 120{,}38\,cm^2 \approx \underline{120\,cm^2}$

$A_2 = A - 2A_1 = \pi(14\,cm)^2 - 2 \cdot 120{,}38\,cm^2 \approx 374{,}99\,cm^2 \approx \underline{370\,cm^2}$

**10.14**




$b = \frac{\alpha}{360°} 2\pi r = \frac{1}{8} \cdot 2\pi r \quad | \cdot \frac{360°}{2\pi r}$

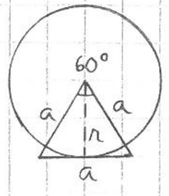
$\Leftrightarrow \alpha = \frac{360°}{8} = 45°$

$A_{seg} = A_s - A_k = \frac{\alpha}{360°}\pi r^2 - \frac{1}{2} \cdot r \cdot r \cdot \sin\alpha$

$= r^2\left(\frac{1}{8}\pi - \frac{1}{2}\sin 45°\right)$

$\frac{A_{seg}}{A_y} = \frac{r^2\left(\frac{1}{8}\pi - \frac{1}{2}\sin 45°\right)}{\pi r^2} = \frac{\frac{\pi}{8} - \frac{1}{2}\sin 45°}{\pi} \approx 0{,}01246 \approx \underline{1{,}2\,\%}$

**10.15**




Pythagoras:

$a^2 = r^2 + \left(\frac{a}{2}\right)^2$

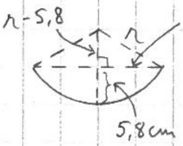
$\Leftrightarrow r^2 = a^2 - \frac{a^2}{4} = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

$\frac{A_m}{A_k} = \frac{A_k - A_s}{A_k} = \frac{A_k}{A_k} - \frac{A_s}{A_k}$

$= 1 - \frac{\frac{60°}{360°} \cdot \pi r^2}{\frac{1}{2} a \cdot a \cdot \sin 60°} = 1 - \frac{\frac{1}{6}\pi \cdot \frac{3a^2}{4}}{\frac{1}{2}a^2 \cdot \frac{\sqrt{3}}{2}} = 1 - \frac{\frac{\pi}{8}}{\frac{\sqrt{3}}{4}}$

$\approx 0{,}0931 \approx \underline{9{,}3\,\%}$

**10.18**




$\frac{18{,}6\,cm}{2} = 9{,}3\,cm$ Pythagoras:

$9{,}3^2 + (r - 5{,}8)^2 = r^2$

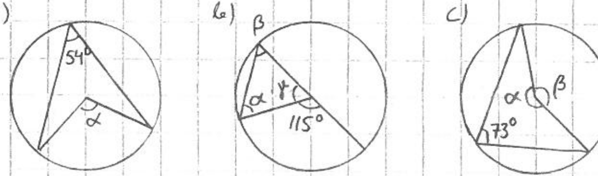
$\Leftrightarrow 9{,}3^2 + r^2 - 11{,}6r + 5{,}8^2 = r^2$

$\Leftrightarrow 9{,}3^2 + 5{,}8^2 = 11{,}6r \quad |:11{,}6$

$\Leftrightarrow r = \frac{9{,}3^2 + 5{,}8^2}{11{,}6} \approx 10{,}356\ (cm)$

$\Rightarrow$ pallon halkaisija: $2r \approx 20{,}712\,cm \approx \underline{20{,}7\,cm}$

**11.1**




a) $\alpha = 2 \cdot 54° = 108°$ (kehäkulmalause)

b) $\beta = \frac{115°}{2} = 57{,}5°$ ( —"— )

$\gamma = 180° - 115° = 65°$

$\alpha = 180° - 57{,}5° - 65° = \underline{57{,}5°}$

TAI: $\alpha = \beta$ (tasakylkisen kolmion kantakulmat)

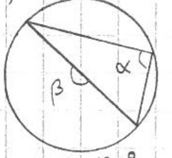
$\gamma = 180° - 115° = 65°$

$\alpha + \beta + \gamma = 2\alpha + 65° = 180° \Leftrightarrow \alpha = \underline{57{,}5°}$

c) $\beta = 2 \cdot 73° = 146°$ (kehäkulmalause)

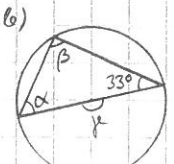
$\alpha = 360° - 146° = \underline{214°}$

**11.4** a)




$\alpha = \frac{\beta}{2} = \frac{180°}{2} = 90°$ (kehäkulmalause)

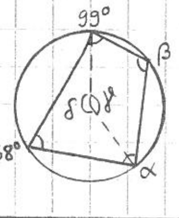
b)




$\beta = \frac{\gamma}{2} = \frac{180°}{2} = 90°$

$\alpha = 180° - 90° - 33° = \underline{57°}$

**11.6**




$\gamma = 2 \cdot 68° = 136°$ (kehäkulmalause)

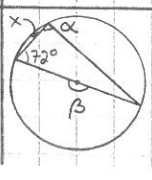
$\delta = 360° - 136° = 224°$

$\beta = \frac{\delta}{2} = \frac{224°}{2} = \underline{112°}$

4-kulmion kulmien summa:

$\alpha + 68° + 99° + 112° = (4-2) \cdot 180° = 360° \Leftrightarrow \underline{\alpha = 8}$

**11.11**




$r = 3{,}0\,cm \qquad \alpha = \frac{\beta}{2} = \frac{180°}{2} = 90°$

$\cos 72° = \frac{x}{2r} \quad \Leftrightarrow x = 2r\cos 72°$

$A = \frac{1}{2} \cdot x \cdot 2r \cdot \sin 72° = \frac{1}{2} \cdot 2r\cos 72° \cdot 2r \cdot \sin 72°$

$\approx 5{,}2901\,cm^2 \approx \underline{5{,}3\,cm^2}$