7. $\bar a = 4\bar i + 25\bar j$, $\bar b = 2\bar i + 3\bar j$, $\bar c = 3\bar i - 5\bar j$

a) $\bar a = x\bar b + y\bar c$

$\Leftrightarrow\ 4\bar i + 25\bar j = x(2\bar i + 3\bar j) + y(3\bar i - 5\bar j)$

$\Leftrightarrow\ 4\bar i + 25\bar j = 2x\bar i + 3x\bar j + 3y\bar i - 5y\bar j$

$\Leftrightarrow\ \underline{4}\bar i + \underline{25}\bar j = \underline{(2x+3y)}\bar i + \underline{(3x-5y)}\bar j$

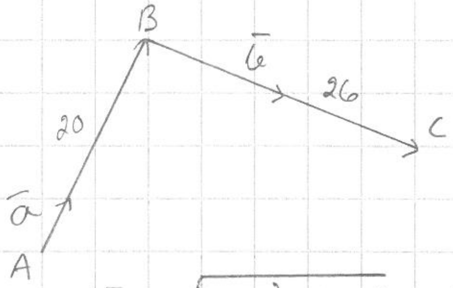
$$\Rightarrow \begin{cases} 4 = 2x + 3y & |\cdot 5 \\ 25 = 3x - 5y & |\cdot 3 \end{cases} \quad \text{yj.}$$

$95 = 19x \quad |:19 \Leftrightarrow x = 5 \quad \Rightarrow 4 = 2\cdot 5 + 3y \Leftrightarrow 3y = -6 \Leftrightarrow y = -2$

$\Rightarrow \underline{\bar a = 5\bar b - 2\bar c}$

b) $\cos(\bar b, \bar c) = \dfrac{\bar b \cdot \bar c}{|\bar b||\bar c|} = \dfrac{2\cdot 3 + 3\cdot(-5)}{\sqrt{2^2+3^2}\cdot\sqrt{3^2+(-5)^2}} = \dfrac{-9}{\sqrt{13}\cdot\sqrt{34}} \Rightarrow \measuredangle(\bar b, \bar c) \approx 115{,}346° \approx \underline{115°}$

8.



$A = (-2,1)$, $\bar a = 4\bar i + 3\bar j$, $\bar b = -12\bar i - 5\bar j$

$|\bar a| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

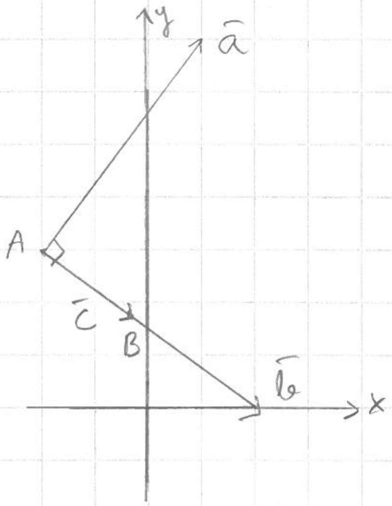
$\overline{AB} = 20\,\bar a^0 = 20\cdot\dfrac{\bar a}{|\bar a|} = 20\cdot\dfrac{\bar a}{5} = 4\bar a = 4(4\bar i + 3\bar j) = 16\bar i + 12\bar j$

$\Rightarrow B = (-2+16, 1+12) = \underline{(14,13)}$

$|\bar b| = \sqrt{(-12)^2+(-5)^2} = \sqrt{169} = 13 \Rightarrow \overline{BC} = 26\,\bar b^0 = 26\cdot\dfrac{\bar b}{|\bar b|} = 26\cdot\dfrac{\bar b}{13} = 2\bar b = 2(-12\bar i - 5\bar j) = -24\bar i - 10\bar j$

$\Rightarrow C = (14-24, 13-10) = \underline{(-10,3)}$

9.



$A = (-2,3)$, $\bar a = 3\bar i + 4\bar j$

a) $\underline{\bar b = 4\bar i - 3\bar j}$

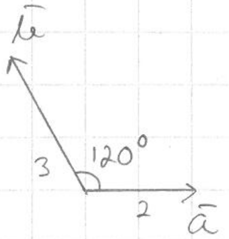
$\bar a \cdot \bar b = 3\cdot 4 + 4\cdot(-3) = 0 \Rightarrow \bar a \perp \bar b$

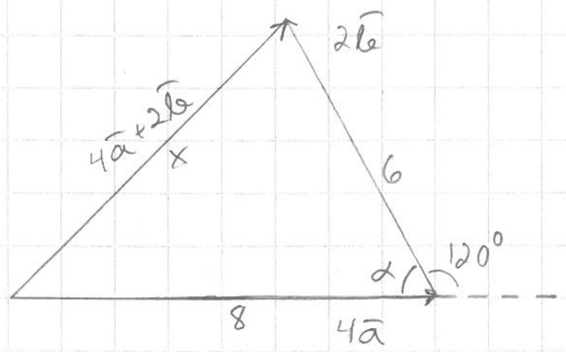
b) $|\bar b| = \sqrt{4^2+(-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$\bar c = 2\bar b^0 = 2\dfrac{\bar b}{|\bar b|} = 2\cdot\dfrac{\bar b}{5} = \dfrac{2}{5}(4\bar i - 3\bar j) = \underline{\dfrac{8}{5}\bar i - \dfrac{6}{5}\bar j}$

c) $B = \left(-2+\dfrac{8}{5}, 3-\dfrac{6}{5}\right) = \underline{\left(-\dfrac{2}{5}, \dfrac{9}{5}\right)}$

10.





$\alpha = 180° - 120° = 60°$

Kosinilause:

$x^2 = 8^2 + 6^2 - 2\cdot 8\cdot 6\cdot\cos 60°$

$= 64 + 36 - 96\cdot\frac{1}{2} = 52$

$\Rightarrow |4\bar a + 2\bar b| = x = \sqrt{52} = \sqrt{4\cdot 13}$

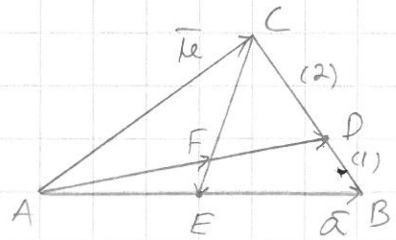
$= \underline{2\sqrt{13}}\ (\approx 7{,}21)$

TAI: $|4\bar a + 2\bar b|^2 = (4\bar a + 2\bar b)\cdot(4\bar a + 2\bar b) = 16\bar a\cdot\bar a + 8\bar a\cdot\bar b + 8\bar a\cdot\bar b + 4\bar b\cdot\bar b$

$= 16|\bar a|^2 + 16\cdot|\bar a||\bar b|\cos(\bar a, \bar b) + 4|\bar b|^2$

$= 16\cdot 2^2 + 16\cdot 2\cdot 3\cdot\cos 120° + 4\cdot 3^2 = 52 \quad \Rightarrow |4\bar a + 2\bar b| = \sqrt{52}$

11.



a) $\overline{AB} = \bar a$, $\overline{AC} = \bar b$

$$\begin{cases} \overline{AD} = \overline{AB} + \overline{BD} = \bar a + \frac{1}{3}\overline{BC} = \bar a + \frac{1}{3}(-\bar a + \bar b) = \frac{2}{3}\bar a + \frac{1}{3}\bar b \\ \overline{AF} = x\overline{AD} = x\left(\frac{2}{3}\bar a + \frac{1}{3}\bar b\right) \end{cases}$$

$$\begin{cases} \overline{CE} = \overline{CA} + \overline{AE} = -\bar b + \frac{1}{2}\bar a \\ \overline{FE} = y\overline{CE} = y\left(-\bar b + \frac{1}{2}\bar a\right) \end{cases}$$

Kierretään $A \to F \to E \to A$:

$\overline{AF} + \overline{FE} + \overline{EA} = \bar 0$

$\Leftrightarrow x\left(\frac{2}{3}\bar a + \frac{1}{3}\bar b\right) + y\left(-\bar b + \frac{1}{2}\bar a\right) - \frac{1}{2}\bar a = \bar 0$

$\Leftrightarrow \underline{\left(\frac{2}{3}x + \frac{1}{2}y - \frac{1}{2}\right)}\bar a + \underline{\left(\frac{1}{3}x - y\right)}\bar b = \bar 0 = \underline{0}\cdot\bar a + \underline{0}\cdot\bar b$

$\bar a \nparallel \bar b$

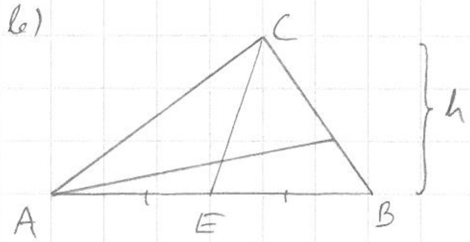
$$\Rightarrow \begin{cases} \frac{2}{3}x + \frac{1}{2}y - \frac{1}{2} = 0 & |\cdot 2 \\ \frac{1}{3}x - y = 0 \end{cases} \quad \text{yj.}$$

$\frac{5}{3}x - 1 = 0 \Leftrightarrow x = \frac{3}{5} \Rightarrow y = \frac{1}{3}\cdot\frac{3}{5} = \frac{1}{5}$

Siten $\dfrac{AF}{FD} = \dfrac{\frac{3}{5}}{\frac{2}{5}} = \dfrac{3}{5}\cdot\dfrac{5}{2} = \dfrac{3}{2} \Rightarrow$ jana AD suhteessa $\underline{3:2}$

$\dfrac{CF}{FE} = \dfrac{\frac{4}{5}}{\frac{1}{5}} = \dfrac{4}{5}\cdot\dfrac{5}{1} = \dfrac{4}{1} \Rightarrow$ jana CE suhteessa $\underline{4:1}$

b)



Kolmioiden AEC ja BEC kannat ovat yhtä pitkät (AE = EB), samoin kolmioiden korkeudet ovat samat (h). Siten kolmioiden pinta-alat ovat yhtä suuret, eli pinta-alojen suhde on $\underline{1:1}$