

16.19

luento: $x, x+1, x+2$

$$\lceil 2 \cdot 3 \cdot 4 = 6 \cdot 4 = 24 \rceil$$

$$\Rightarrow x(x+1)(x+2) = x + (x+1) + (x+2)$$

$$\Leftrightarrow (x^2+x)(x+2) = 3x+3$$

$$\Leftrightarrow x^3 + 2x^2 + x^2 + 2x = 3x + 3$$

$$\Leftrightarrow x^3 + 3x^2 - x - 3 = 0$$

$$\Leftrightarrow x^2(x+3) - (x+3) = 0 \quad (\text{ryhmittelyperia})$$

$$\Leftrightarrow (x+3)(x^2-1) = 0$$

$$\Leftrightarrow x+3=0 \quad \text{tai} \quad x^2-1=0$$

$$\Leftrightarrow x = -3$$

$$x^2=1 \quad \sqrt{\quad} \Leftrightarrow x = \pm 1$$

$$x = -3 = \underline{\underline{-3, -2, -1}}$$

$$x = -1 = \underline{\underline{-1, 0, 1}}$$

$$x = 1 = \underline{\underline{1, 2, 3}}$$

Esim.

$$x^4 + 3x^2 - 4 = 0 \quad \Leftrightarrow (x^2)^2 + 3x^2 - 4 = 0$$

$$(x^2)^2$$

$$\text{spuunnuttaja: } x^2 = t \Rightarrow t^2 + 3t - 4 = 0 \quad \Leftrightarrow t = \begin{cases} 1 \\ -4 \end{cases}$$

$$\Rightarrow x^2 = 1 \quad \sqrt{\quad} \quad \text{tai} \quad x^2 = -4 \quad \downarrow$$

$$\Leftrightarrow x = \pm 1$$

16.8 a)

$$4x^2 + 4 = 3x^3 + 3x$$

$$\Leftrightarrow -3x^3 + 4x^2 - 3x + 4 = 0$$

$$\Leftrightarrow x^2(-3x+4) + (-3x+4) = 0 \quad (\text{ryhmittelyperia})$$

$$\Leftrightarrow (-3x+4)(x^2+1) = 0$$

$$\Leftrightarrow -3x+4=0 \quad \text{tai} \quad x^2+1=0$$

$$\Leftrightarrow x = \underline{\underline{\frac{4}{3}}}$$

$$\Leftrightarrow x^2 = -1 \quad \downarrow$$

17. Nollakohtien ja tekijöiden yhteys

$$\text{Esim. } 2x^2 - 8x + 6 = 0 \quad \Leftrightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2} = \frac{8 \pm \sqrt{64 - 48}}{4} = \frac{8 \pm \sqrt{16}}{4} = \frac{8 \pm 4}{4}$$

$$= \begin{cases} 3 \\ 1 \end{cases}$$

0-kohtat

$$2(x-3)(x-1) = (2x-6)(x-1) = 2x^2 - 2x - 6x + 6 = 2x^2 - 8x + 6$$

 $\uparrow \quad \uparrow$
tekijät

$$\text{yleisesti } 1^\circ D > 0 : ax^2 + bx + c = 0 \quad \Leftrightarrow x = \begin{cases} x_1 \\ x_2 \end{cases}$$

$$\Rightarrow ax^2 + bx + c = a(x-x_1)(x-x_2)$$

$$2^\circ D = 0 : ax^2 + bx + c = 0 \quad \Leftrightarrow x = x_1 \quad (\text{kaksoisjuuri})$$