4.18 a) $\underbrace{(7x+6)(7x-6)}_{(7x)^2-6^2}(49x^2+36) = (49x^2-36)(49x^2+36)$

$= (49x^2)^2 - 36^2 = 2401x^4 - 1296$

b) $(3x^2-4)^2 - 3(x^2-4)(x^2+4) = \left((3x^2)^2 - 2\cdot 3x^2\cdot 4 + 4^2\right) - 3\left((x^2)^2 - 4^2\right)$

$= 9x^4 - 24x^2 + 16 - 3x^4 + 48 = 6x^4 - 24x^2 + 64$

4.19 $5^3 - 5 = 120 = 4\cdot 5\cdot 6$

$1^3 - 1 = 0 = 0\cdot 1\cdot 2$ ✓

$2^3 - 2 = 8-2 = 6 = 1\cdot 2\cdot 3$ ✓

$3^3 - 3 = 27-3 = 24 = 2\cdot 3\cdot 4$ ✓

⋮

Ehkä kaava pätee aina

Yleisesti $n^3 - n = (n-1)\,n\,(n+1)$

$= (n^2-1^2)\,n$

$= (n^2-1)\,n$

$= n^3 - n$ ✓ $\Rightarrow$ on

Esim. Ratkaise yhtälö $x^2 - y^2 = 15$ kun $x, y \in \mathbb{N} = \{0, 1, 2, \ldots\}$

Ratk. $x^2 - y^2 = 15$

$\Leftrightarrow \underbrace{(x-y)}_{\in\mathbb{N}}\underbrace{(x+y)}_{\in\mathbb{N}} = 15 = 3\cdot 5 = 1\cdot 15$

$\Rightarrow \begin{cases} x-y = 3 \\ x+y = 5 \end{cases}$ ←

$2x = 8 \quad | :2 \Leftrightarrow x = 4$

$y = 5 - x = 5 - 4 = 1$

Tark. $4^2 - 1^2 = 15$ ✓

tai $\begin{cases} x-y = 1 \\ x+y = 15 \end{cases}$ ← myös

$2x = 16 \quad | :2 \Leftrightarrow x = 8$

$y = x - 1 = 8 - 1 = 7$

Tark. $8^2 - 7^2 = 15$ ✓

5. Tekijöihin jakaminen

Esim. a) $5x^2 - 10x = 5x(x - 2)$

b) $4x^3 - 16x = 4x(x^2 - 4) = 4x(x^2 - 2^2) = 4x(x-2)(x+2)$

c)

d)