MAA PRELI 31.1.2023

1. a) $2x^2-7x-4=0 \iff x=\frac{7\pm\sqrt{(-7)^2-4\cdot2\cdot(-4)}}{2\cdot2}=\frac{7\pm\sqrt{49+32}}{4}=\frac{7\pm\sqrt{81}}{4}=\frac{7\pm9}{4}=\begin{cases}4\\-\frac12\end{cases}$

b) $(x+a)^2=x^2+14x+b \iff x^2+2ax+a^2=x^2+14x+b$

$\Rightarrow\begin{cases}1=1\\2a=14 \iff a=7\\a^2=b \Rightarrow b=7^2=49\end{cases}$

c) $p(x)=cx+d \Rightarrow \begin{cases}p(4)=4c+d=1 & |\cdot(-1)\\ p(7)=7c+d=3\end{cases}$

$3c=2 \iff c=\frac23 \Rightarrow d=1-4\cdot\frac23=-\frac53$

$p(x)=\frac23x-\frac53=0 \ |\cdot3 \iff 2x-5=0 \iff x=\frac52$

2. 1. kuva: D, 2. kuva: A, 3. kuva: B, 4. kuva: D, 5. kuva: C, 6. kuva: E

3. $\bar a=\bar i+2\bar j+3\bar k$, $\bar b=2\bar i+5\bar k$, $\bar c_t=t\bar a+(1-t)\bar b$, $-2\le t\le2$

$\bar c_t=t(\bar i+2\bar j+3\bar k)+(1-t)(2\bar i+5\bar k)=(2-t)\bar i+2t\bar j+(5-2t)\bar k$

$|\bar c_t|=\sqrt{(2-t)^2+(2t)^2+(5-2t)^2}=\sqrt{4-4t+t^2+4t^2+25-20t+4t^2}$

$=\sqrt{\underbrace{9t^2-24t+29}_{=f(t)\ge0}}$, $\sqrt x$ aidosti kasvava funktio $\Rightarrow |\bar c_t|$ pienin kun $f(t)$ pienin, f jatk. ja deriv. väl. $[-2,2]$

$f'(t)=18t-24=0 \iff t=\frac{24}{18}=\frac{12}{9}=\frac43$

	-2		$\frac43$		2
f'		$-$		$+$	
f		↘	min	↗	

Vast. $t=\frac43$

4. a) $x,y>0$ $\quad \log_4 y=\log_2 x \iff \frac{\ln y}{\ln 4}=\frac{\ln x}{\ln 2} \iff \frac{\ln y}{\ln 2^2}=\frac{\ln x}{\ln 2}$

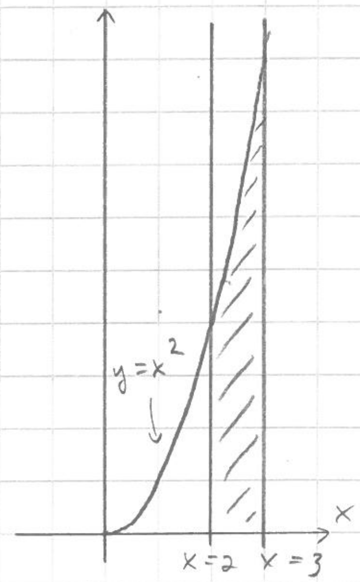
$\iff \frac{\ln y}{2\ln 2}=\frac{\ln x}{\ln 2} \ |\cdot 2\ln 2 \iff \ln y=2\ln x=\ln x^2 \ |e^{()} \iff y=x^2$

⌈TAI: $\log_4 y=\log_2 x \iff 4^{\log_2 x}=y \iff y=(2^2)^{\log_2 x}$

$\iff y=2^{2\log_2 x}=2^{\log_2 x^2}=x^2$⌋

b) $A=\int_2^3 x^2\,dx=\Big/_2^3 \frac13x^3=\frac13\cdot3^3-\frac13\cdot2^3=\frac13(27-8)$

$=\frac{19}{3}\ (\approx 6{,}33)$



11. a) $P(7\%)=\frac{7}{40}\cdot\frac{6}{39}\cdot\frac{5}{38}\cdot\frac{4}{37}\cdot\frac{3}{36}\cdot\frac{2}{35}\cdot\frac{1}{34}=\frac{1}{18\,643\,560}\approx5{,}36\cdot10^{-8}$

⌈TAI: $\frac{1}{\binom{40}{7}}=\frac{1}{18\,643\,560}\approx5{,}36\cdot10^{-8}$⌋

b) $P(6\%)=\underbrace{\frac{7}{40}\cdot\frac{6}{39}\cdot\frac{5}{38}\cdot\frac{4}{37}\cdot\frac{3}{36}\cdot\frac{2}{35}}_{6\text{ kpl }\%}\cdot\underbrace{\frac{32}{34}}_{\text{väärä}}\cdot\underbrace{7}_{\text{eri järj.}}\approx1{,}20\cdot10^{-5}$

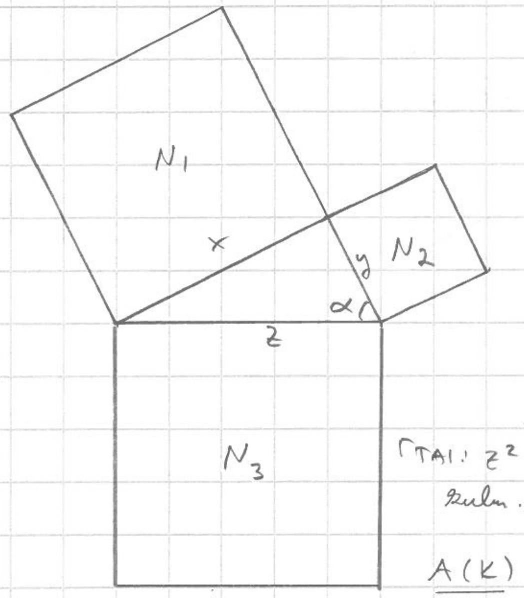
⌈TAI: $\frac{\binom{7}{6}\binom{32}{1}}{\binom{40}{7}}\approx1{,}20\cdot10^{-5}$⌋

c) $P(3\%+\text{lisä})=\underbrace{\frac{7}{40}\cdot\frac{6}{39}\cdot\frac{5}{38}}_{3\text{ kpl }\%}\cdot\underbrace{\frac{1}{37}}_{\text{lisä}}\cdot\underbrace{\frac{32}{36}\cdot\frac{31}{35}\cdot\frac{30}{34}}_{3\text{ kpl väärä}}\cdot\underbrace{\binom{7}{3}\cdot\binom{4}{1}}_{\text{eri järj.}}\approx0{,}00931$

⌈TAI: $\frac{\binom{7}{3}\binom{1}{1}\binom{32}{3}}{\binom{40}{7}}=\frac{4340}{466089}\approx0{,}00931$⌋

d) Sitä sopii miettiä!

12. $A(N_1):A(N_2):A(N_3)=9:2:11 \Rightarrow A(N_1)=9a^2,\ A(N_2)=2a^2,\ A(N_3)=11a^2$



$x^2=A(N_1)=9a^2 \ |\sqrt{\ } \iff x=\sqrt{9a^2}=\sqrt9|a|=\sqrt9a$

Vastaavasti $y=\sqrt2a$, $z=\sqrt{11}a$

Kosinilause: $x^2=y^2+z^2-2\cdot y\cdot z\cdot\cos\alpha$

$\iff \cos\alpha=\frac{x^2-y^2-z^2}{-2yz}=\frac{9a^2-2a^2-11a^2}{-2\cdot\sqrt2a\sqrt{11}a}$

$=\frac{-4a^2}{-2\sqrt{22}a^2}=\frac{2}{\sqrt{22}}=\frac{2\sqrt{22}}{22}=\frac{\sqrt{22}}{11}$

$\sin^2\alpha+\cos^2\alpha=1 \iff \sin\alpha=(\pm)\sqrt{1-\cos^2\alpha}=\sqrt{1-\left(\frac{\sqrt{22}}{11}\right)^2}$ $\quad 0°<\alpha<180°$

⌈TAI: $z^2=x^2+y^2 \Rightarrow$ suora-kulm. kolmio $\Rightarrow A(K)=\frac12yx\ \frac12 3\sqrt2a^2$⌋

$=\sqrt{1-\frac{22}{121}}=\sqrt{\frac{9}{11}}=\frac{3}{\sqrt{11}}$

$\frac{A(K)}{A(N_2)}=\frac{\frac12\cdot yz\sin\alpha}{2a^2}=\frac{\frac12\cdot\sqrt2a\sqrt{11}a\cdot\frac{3}{\sqrt{11}}}{2a^2}=\frac{3\sqrt2}{4}\ (\approx1{,}06)$

13. $f(x)=\sin\frac1x$, $x\ne0$

a) $a_n=\frac{1}{n\pi}$, $k=1,2,3,\ldots \Rightarrow \begin{cases}\lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{1}{\underbrace{n\pi}_{\to\infty}}=0\\ \lim\limits_{n\to\infty}f(a_n)=\lim\limits_{n\to\infty}\sin\left(\frac{1}{\frac{1}{n\pi}}\right)=\lim\limits_{n\to\infty}\underbrace{\sin n\pi}_{=0}=0\end{cases}$

b) $a_n=\frac{1}{\underbrace{t+n2\pi}_{-\frac\pi2\le t\le\frac\pi2,\ \ge2\pi}}$, $-\frac\pi2\le t\le\frac\pi2$, $n=1,2,\ldots$

1° Selvästi $a_n>0$ aina koska $t+n2\pi\ge-\frac\pi2+2\pi=\frac{3\pi}{2}>0$

2° $\lim\limits_{n\to\infty}a_n=\lim\limits_{n\to\infty}\frac{1}{\underbrace{t+n2\pi}_{\to\infty}}=0$

3° $\lim\limits_{n\to\infty}f(a_n)=\lim\limits_{n\to\infty}\sin\frac{1}{a_n}=\lim\limits_{n\to\infty}\sin\frac{1}{\frac{1}{t+n2\pi}}=\lim\limits_{n\to\infty}\sin(t+n2\pi)$

$=\lim\limits_{n\to\infty}\sin t=\sin t$