7.1

a) $\sum_{k=1}^{\infty} \underbrace{\frac{k}{k+10}}_{a_k} = \frac{1}{1+10} + \frac{2}{2+10} + \frac{3}{3+10} + \dots$

$\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{k}{k+10} \overset{(\frac{\infty}{\infty})}{=} \lim_{k\to\infty} \frac{\cancel{k}}{\cancel{k}(1+\frac{10}{k})} = \lim_{k\to\infty} \frac{1}{1+\frac{10}{k}} = \frac{1}{1+0} = 1 \neq 0$

$\Rightarrow$ sarja hajaantuu

b) $\sum_{k=1}^{\infty} \underbrace{\left(\frac{1}{2k-1} - \frac{1}{2k+1}\right)}_{=a_k} = \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \dots$

$a_k = \frac{1}{2k-1} - \frac{1}{2k+1} \xrightarrow{k\to\infty} \frac{1}{\infty} - \frac{1}{\infty} = 0 - 0 = 0$

$\Rightarrow$ ei voi päätellä suppenemisesta mitään

Sarjan n. osasumma

$S_n = \sum_{k=1}^{n} \left(\frac{1}{2k-1} - \frac{1}{2k+1}\right) = \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \dots$

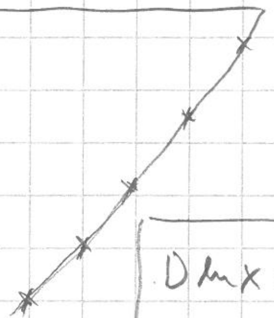
$+ \left(\cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}\right)$

$= 1 - \frac{1}{2n+1} \xrightarrow{n\to\infty} 1 - \frac{1}{\infty} = 1 - 0 = 1$

teleskooppi-ilmiö

$\Rightarrow$ sarja suppenee ja sen arvo $= 1$

8. $a_n = \ln \frac{3n+1}{n+4}$, $n = 1, 2, 3, \dots$



a) $f(x) = \ln \frac{3x+1}{x+4}$, f on jatk. ja deriv. kun $x \geq 1$

ja $f(n) = a_n$

$\boxed{D \ln x = \frac{1}{x} \quad D \ln f(x) = \frac{1}{f(x)} \cdot f'(x)}$

$f'(x) = \frac{1}{\frac{3x+1}{x+4}} \cdot \frac{3(x+4) - (3x+1)\cdot 1}{(x+4)^2}$

$= \frac{\overbrace{x+4}^{>0}}{\underbrace{3x+1}_{>0}} \cdot \frac{\overbrace{11}^{>0}}{\underbrace{(x+4)^2}_{>0}} > 0$ aina kun $x \geq 1$

$\Rightarrow$ f aidosti kasvava kun $x \geq 1$ $\Rightarrow$ (a_n) aid. kasvava

b) $a_n = \ln \frac{3n+1}{n+4} \overset{(\ln \frac{\infty}{\infty})}{=} \ln \frac{\cancel{n}(3+\frac{1}{n})}{\cancel{n}(1+\frac{4}{n})} \xrightarrow{n\to\infty} \ln \frac{3+0}{1+0} = \ln 3$

c) $|a_n - \ln 3| = \left| \ln \frac{3n+1}{n+4} - \ln 3 \right| < 0{,}0001$ $\overset{\text{laskimella}}{\Rightarrow}$ $n > 36\,664{,}5$

$\Rightarrow$ arvosta 36 665 alkaen