

$$2. \quad a) \quad \lim_{m \rightarrow \infty} \frac{3m^2 - 2m}{m^2 + 5m} \stackrel{\left(\frac{\infty - \infty}{\infty}\right)}{=} \lim_{m \rightarrow \infty} \frac{m^2 \left(3 - \frac{2}{m}\right)}{m^2 \left(1 + \frac{5}{m}\right)} = \lim_{m \rightarrow \infty} \frac{3 - \left(\frac{2}{m}\right) \rightarrow 0}{1 + \left(\frac{5}{m}\right) \rightarrow 0}$$

$$= \frac{3 - 0}{1 + 0} = \frac{3}{1} = \underline{3}$$

$$b) \quad f(x, y) = xy + x^2$$

Orittaiderivotta muuttujan  $x$  suhteen:  $f'_x(x, y) = y + 2x$

$y$  - suhteen:  $f'_y(x, y) = x$

$$\begin{cases} f'_x(1, 2) = 2 + 2 \cdot 1 = \underline{4} \\ f'_y(1, 2) = \underline{1} \end{cases}$$

$$3. \quad a) \quad 100 - 110 + 121 - \dots$$

$$\left. \begin{aligned} q &= \frac{-110}{100} = -1,1 \\ q &= \frac{121}{-110} = -1,1 \end{aligned} \right\} \Rightarrow |q| = |-1,1| = 1,1 < 1 \quad \downarrow$$

$\Rightarrow$  sarja kajaantuu

$$b) \quad 9 + 3 + 1 + \dots = \frac{9}{1 - \frac{1}{3}} = \frac{9}{\frac{2}{3}} = \frac{9 \cdot 3}{2} = \underline{\frac{27}{2}}$$

$$q = \frac{3}{9} = \frac{1}{3}$$

$$q = \frac{1}{3}$$

$$|q| = \left|\frac{1}{3}\right| = \frac{1}{3} < 1 \quad \Rightarrow \text{sarja suppenee}$$

$$4. \quad a) \quad \int_1^{\infty} \frac{1}{x} dx = I$$

$$\int_1^t \frac{1}{x} dx = \int_1^t \ln|x| = \ln t - \underbrace{\ln 1}_{=0} = \ln t \xrightarrow{t \rightarrow \infty} \ln \infty = \infty$$

$\Rightarrow I$  kajaantuu

$$b) \quad \int_0^1 \frac{1}{\sqrt{x}} dx = I$$

äimäntälty 0:lle  $\Rightarrow$  epäoleellinen integraali

$$\begin{aligned} \int_{\frac{1}{n}}^1 \frac{1}{\sqrt{x}} dx &= \int_{\frac{1}{n}}^1 \frac{1}{x^{\frac{1}{2}}} dx = \int_{\frac{1}{n}}^1 x^{-\frac{1}{2}} dx = \left[ \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} \right]_{\frac{1}{n}}^1 \\ &= \left[ 2\sqrt{x} \right]_{\frac{1}{n}}^1 = 2\sqrt{1} - 2\sqrt{\frac{1}{n}} \xrightarrow{n \rightarrow \infty^+} 2 - 2\sqrt{0} = 2 = \underline{I} \end{aligned}$$

$$5. \quad f(x) = \frac{4x^2 - 3x}{x^2 + 1}$$

$\underbrace{x^2 + 1}_{\geq 0}$

,  $f$  jalk. ja derivo.  $\mathbb{R}$ :ssä

a)  $x \in \mathbb{R}$