

$$\Rightarrow B = \left(1 - \frac{12}{5}, 2 + \frac{9}{5}\right) = \left(-\frac{7}{5}, \frac{19}{5}\right)$$

$$\text{tai } \overline{AB} = \frac{3}{5}(-\overline{b}) = \frac{12}{5}\overline{i} - \frac{9}{5}\overline{j}$$

$$\Rightarrow B = \left(1 + \frac{12}{5}, 2 - \frac{9}{5}\right) = \left(\frac{17}{5}, \frac{1}{5}\right)$$

10. Rajo-arvo, jatkuvuus ja derivaatta

10.1 a) $\lim_{x \rightarrow 5} \frac{3x-15}{x^2+25} = \frac{3 \cdot 5 - 15}{5^2 + 25} = \frac{0}{50} = 0$

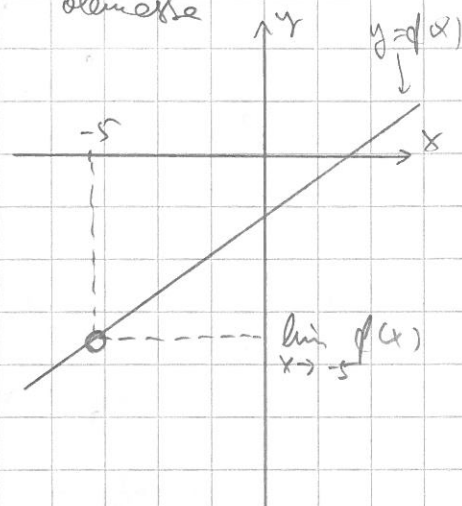
b) $\lim_{x \rightarrow 5} \frac{3x-15}{x^2-25} = \left(\frac{0}{0}\right) = \lim_{x \rightarrow 5} \frac{3(x-5)}{(x-5)(x+5)} = \lim_{x \rightarrow 5} \frac{3}{x+5} = \frac{3}{5+5} = \frac{3}{10}$

c) $\lim_{x \rightarrow 5} \frac{3x-15}{(x-5)^2} = \left(\frac{0}{0}\right) = \lim_{x \rightarrow 5} \frac{3(x-5)}{(x-5)^2} = \frac{3}{0} = \pm\infty \Rightarrow$ rajo-arvoa ei ole olemassa

10.3

10.4 $f(x) = \frac{x^2-25}{3x+15}, x \neq -5$

$$\begin{aligned} \lim_{x \rightarrow -5} f(x) &= \lim_{x \rightarrow -5} \frac{x^2-25}{3x+15} = \left(\frac{0}{0}\right) = \lim_{x \rightarrow -5} \frac{(x-5)(x+5)}{3(x+5)} \\ &= \lim_{x \rightarrow -5} \frac{x-5}{3} = \frac{-5-5}{3} = -\frac{10}{3} \end{aligned}$$



f jatkuu kohdassa $-5 \Leftrightarrow f(-5) = \lim_{x \rightarrow -5} f(x) = -\frac{10}{3}$

Siis

$$f(x) = \begin{cases} \frac{x^2-25}{3x+15}, & x \neq -5 \\ -\frac{10}{3}, & x = -5 \end{cases}$$

10.11

$$2x^3 + ax^2 + a^2x + 1 = 0$$

$= f(x)$, f jatk. ja deriva. $\mathbb{R} : \mathbb{R}$

$f(0) = 1 > 0$

$f(-1) = 2 \cdot (-1)^3 + a \cdot (-1)^2 + a^2 \cdot (-1) + 1 = -2 + a - a^2 + 1 = -a^2 + a - 1 < 0$ (*)

f jatkuu $\mathbb{R} : \mathbb{R}$

$\Rightarrow f$:llä on ainakin 10-kolme välillä $]-1, 0[\Rightarrow$ välillä on ainakin 1 juuri välillä $]-1, 0[$

(*) $-a^2 + a - 1 = 0$, $D = 1^2 - 4 \cdot (-1) \cdot (-1) = 1 - 4 = -3 < 0 \Rightarrow$ ei ratk.

$\hookrightarrow -a^2 + a - 1 < 0$ aina]