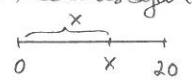


13.8 X : matkustajan odotusaika (0...20)

a)  $0 < x < 20 : F(x) = P(X \leq x) = \frac{x}{20}$

$\Rightarrow F(x) = \begin{cases} 0 & , x \leq 0 \\ \frac{x}{20} & , 0 < x < 20 \\ 1 & , x \geq 20 \end{cases}$

b) $P(5 \leq X \leq 10) = P(X \leq 10) - P(X < 5) = F(10) - F(5)$
 $= \frac{10}{20} - \frac{5}{20} = \frac{5}{20} = \underline{0,25}$

14.17 X : vilkkaan kantokuoli, $X \sim N(2027, 15)$

$P(\text{vilkkaan kesto} \geq 2000 \text{ kg}) = P(X \geq 2000) = 0,9641$

$P(\text{ketju kesto} \geq 2000 \text{ kg}) = P(10 \text{ vilkkaan kesto} \geq 2000)$
 $= 0,9641^{10} \approx \underline{0,694}$

15.1 $Z \sim N(0,1)$

a) $P(Z \leq 1,5) \approx \underline{0,9332}$, b) $P(Z \geq 0,92) \approx \underline{0,179}$

c) $P(-0,6 \leq Z \leq 1,2) \approx \underline{0,611}$

13.11 a) $f(x) = \begin{cases} kx & , 1 \leq x \leq 5 \\ 0 & , \text{muualla} \end{cases}$

1° $f(x) \geq 0 : kx \geq 0 \Leftrightarrow k \geq 0$

2° $\int_{-\infty}^{\infty} f(x) dx = \int_1^5 kx dx = \left[\frac{k}{2} x^2 \right]_1^5 = \frac{k}{2} (5^2 - 1^2)$
 $= \frac{k}{2} \cdot 24 = 12k = 1 \Leftrightarrow k = \frac{1}{12}$

1° ja 2° $\Rightarrow k = \frac{1}{12}$

b) $1 < x < 5 : F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt = \int_1^x \frac{1}{12} t dt$
 $= \left[\frac{1}{24} t^2 \right]_1^x = \frac{1}{24} x^2 - \frac{1}{24} \cdot 1^2 = \frac{1}{24} x^2 - \frac{1}{24}$

$\Rightarrow F(x) = \begin{cases} 0 & , x \leq 1 \\ \frac{1}{24} x^2 - \frac{1}{24} & , 1 < x < 5 \\ 1 & , x \geq 5 \end{cases}$

15.5 $X \sim N(28, \sigma)$

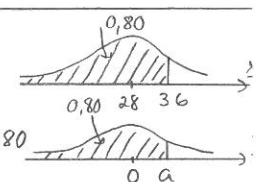
$P(X \leq 36) = 0,80$

$\Leftrightarrow P\left(\frac{X-28}{\sigma} \leq \frac{36-28}{\sigma}\right) = P(Z \leq a) = 0,80$

$Z \sim N(0,1) = a$

$\Rightarrow a = \frac{36-28}{\sigma} \approx 0,8416 \quad | \cdot \frac{\sigma}{0,8416}$

$\Leftrightarrow \sigma = \frac{36-28}{0,8416} \approx 9,5055 \approx \underline{9,5}$



15.8 X : 7-vuotiaan pojan pituus, $X \sim N(\mu, \sigma)$

1° $P(X \leq 125) = \frac{1}{2} \Rightarrow \mu = \underline{125}$

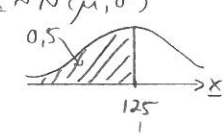
2° $P(X > 132) = 0,07$

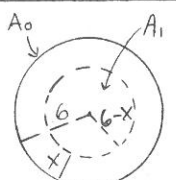
$\Leftrightarrow P\left(\frac{X-125}{\sigma} > \frac{132-125}{\sigma}\right) = P(Z > a) = 0,07$

$Z \sim N(0,1) = a$

$\Rightarrow a = \frac{132-125}{\sigma} \approx 1,4758 \quad | \cdot \frac{\sigma}{1,4758}$

$\Leftrightarrow \sigma = \frac{132-125}{1,4758} \approx 4,7432 \approx \underline{5 \text{ (cm)}}$



13.13  halkaisija: 12,0 m

X : reunen etäisyys reunasta (0...6)

a) $0 < x < 6 : F(x) = P(X \leq x) = \frac{A_0 - A_1}{A_0}$

$= \frac{A_0 - \pi x^2}{A_0} = 1 - \frac{x^2}{6^2} = 1 - \left(\frac{x}{6}\right)^2$

$= 1 - \left(1 - \frac{x}{6}\right)^2 = 1 - \left(1 - \frac{x}{3} + \frac{x^2}{36}\right) = \frac{x}{3} - \frac{x^2}{36}$

$\Rightarrow F(x) = \begin{cases} 0 & , x \leq 0 \\ \frac{x}{3} - \frac{x^2}{36} & , 0 < x < 6 \\ 1 & , x \geq 6 \end{cases}$

b) $P(X > 3) = 1 - P(X \leq 3) = 1 - F(3) = 1 - \left(\frac{3}{3} - \frac{3^2}{36}\right)$
 $= \underline{\frac{1}{4} = 0,25}$

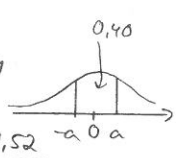
15.12 $Z \sim N(0,1)$

a) $P(Z \leq a) = 0,95 \Rightarrow \underline{a = 1,64}$

b) $P(Z \geq a) = 0,80 \Rightarrow \underline{a = -0,84}$

c) $P(-a \leq Z \leq a) = 0,40$

$\Leftrightarrow P(Z \geq a) = \frac{1-0,40}{2} = 0,30 \Rightarrow \underline{a = 0,52}$



14.1 X : suomalaisten 4-vuotiaiden poikien pituus (cm)

$X \sim N(105, 4)$

a) $P(97 \leq X \leq 105) = 0,14 + 0,34 = \underline{0,48}$

b) $P(X \leq 105) = 0,02 + 0,14 + 0,34 + 0,34 = \underline{0,84}$

c) $P(X > 109) = 0,14 + 0,02 = \underline{0,16}$

d) $P(X = 105) = \underline{0}$

15.16 X : lampun käyttöikä (h), $X \sim N(\mu, \sigma)$

$P(X < 28000) = P\left(\frac{X-\mu}{\sigma} < \frac{28000-\mu}{\sigma}\right) = P(Z < a) = \frac{400}{1000} = 0,4$

$Z \sim N(0,1) = a \Rightarrow a = -0,2533$

$P(X > 34000) = P\left(\frac{X-\mu}{\sigma} > \frac{34000-\mu}{\sigma}\right) = P(Z > b) = \frac{50}{1000} = 0,05$

$Z \sim N(0,1) = b \Rightarrow b = 1,6449$

$\begin{cases} \frac{28000-\mu}{\sigma} = a & \Leftrightarrow \sigma = \frac{28000-\mu}{a} \\ \frac{34000-\mu}{\sigma} = b & \Leftrightarrow \sigma = \frac{34000-\mu}{b} \end{cases}$

$\Rightarrow \frac{28000-\mu}{a} = \frac{34000-\mu}{b} \quad | \cdot \frac{ab}{b}$ $\Leftrightarrow b(28000-\mu) = a(34000-\mu)$

$\Leftrightarrow \mu(a-b) = a \cdot 34000 - b \cdot 28000$

$\Leftrightarrow \underline{\mu = \frac{a \cdot 34000 - b \cdot 28000}{a-b} \approx 28800,65 \approx 28800 \text{ (h)}}$

$\sigma = \frac{28000 - 28800,65}{-0,2533} \approx 3160,88 \approx \underline{3160 \text{ (h)}}$

14.2 $X \sim N(505, 3; 4,1)$

a) $P(X < 500) \approx 0,09806 \approx \underline{0,098}$

b) $P(500 \leq X \leq 515) = 0,893 \approx \underline{89\%}$

14.4 X : 12-vuotiaiden tyttöjen pituus, $X \sim N(153,1; 6,9)$

a) $P(X \leq a) = 0,1 \Rightarrow a \approx 149,26 \Rightarrow \underline{\text{alle } 149 \text{ cm}}$

b) $P(X \geq b) = 0,05 \Rightarrow b \approx 164,45 \Rightarrow \underline{\text{yli } 164 \text{ cm}}$

14.10 $X \sim N(180,7; 6)$ (miehet)

$Y \sim N(167,5; 5,4)$ (naiset)

a) $P(X \geq 190) \approx \underline{0,0606}$

b) $P(Y < 162) \approx 0,154 \approx \underline{15\%}$

c) $P(Y > L) = 0,04 \Leftrightarrow L = 176,95 \approx \underline{177 \text{ (cm)}}$

16.1 X : tulosten "1" lkm 30 nojapöydällä

a) $X \sim \text{Bin}\left(30, \frac{1}{6}\right)$

$P(X = 9) = \binom{30}{9} \left(\frac{1}{6}\right)^9 \left(\frac{5}{6}\right)^{21} \approx 0,0309$

b) $\mu = np = 30 \cdot \frac{1}{6} = 5$

$\sigma = \sqrt{np(1-p)} = \sqrt{30 \cdot \frac{1}{6} \cdot \frac{5}{6}} = \frac{\sqrt{16}}{6} \Rightarrow X \sim N\left(5, \frac{\sqrt{16}}{6}\right)$ likimain

$P(X = 9) = P(8,5 \leq X \leq 9,5) \approx \underline{0,0295}$

sittämääräkorjaus