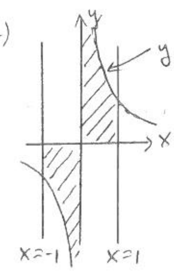
b) 

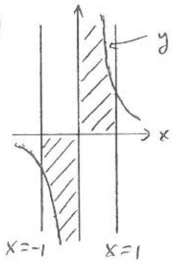
$y = f = \frac{1}{x}$, $x \neq 0$

$A = \int_{-1}^{1} |f(x)|\,dx = \int_{-1}^{1} \left|\frac{1}{x}\right| dx = \int_{-1}^{0} -\frac{1}{x}\,dx + \int_{0}^{1} \frac{1}{x}\,dx$
$= I_1 + I_2$

$\int_{-1}^{t} -\frac{1}{x}\,dx = \Big/_{-1}^{t} -\ln|x| = -\ln(-t) - (\underbrace{-\ln 1}_{=0})$

$= -\ln(-t) \xrightarrow{t \to 0^-} -\ln 0 = -(-\infty) = \infty$

$\Rightarrow I_1$ hajaantuu $\Rightarrow$ A on ∞:n iso

c) 

$y = f(x) = \frac{1}{x^3}$, $x \neq 0$

$A = \int_{-1}^{1} |f(x)|\,dx = \int_{-1}^{1} \left|\frac{1}{x^3}\right| dx$
$= \int_{-1}^{0} -\frac{1}{x^3}\,dx + \int_{0}^{1} \frac{1}{x^3}\,dx = I_1 + I_2$

$\int_{-1}^{t} -\frac{1}{x^3}\,dx = -\int_{-1}^{t} x^{-3}\,dx = -\Big/_{-1}^{t} \frac{1}{-2} x^{-2}$

$= \frac{1}{2}\left[\frac{1}{t^2} - \frac{1}{(-1)^2}\right] \xrightarrow{t \to 0^-} \frac{1}{2}[\infty - 1] = \infty$

$\Rightarrow I_1$ hajaantuu $\Rightarrow$ A on ∞:n iso

11.7 $f(x) = \frac{1}{\sqrt[3]{x^2}}$, $x \neq 0$

$A = A_1 + A_2 = \int_{-8}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

$\int_{-8}^{t} f(x)\,dx = \int_{-8}^{t} \frac{1}{\sqrt[3]{x^2}}\,dx = \int_{-8}^{t} x^{-\frac{2}{3}}\,dx = \Big/_{-8}^{t} \frac{1}{\frac{1}{3}} x^{\frac{1}{3}}$

$= 3\left[\sqrt[3]{t} - \sqrt[3]{-8}\right] \xrightarrow{t \to 0^-} 3[0 - (-2)] = 6 = A_1$

$\int_{s}^{1} f(x)\,dx = \int_{s}^{1} \frac{1}{\sqrt[3]{x^2}}\,dx = \ldots = 3\left[\sqrt[3]{1} - \sqrt[3]{s}\right] \xrightarrow{s \to 0^+} 3[1 - 0] = 3 = A_2$

$\Rightarrow A = A_1 + A_2 = 6 + 3 = \underline{9}$

11.10 

$y = f(x) = e^{-|x|} > 0$

$A = \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} e^{x}\,dx + \int_{0}^{\infty} e^{-x}\,dx = A_1 + A_2$

$\int_{t}^{0} e^{x}\,dx = \Big/_{t}^{0} e^{x} = e^{0} - e^{t} \xrightarrow{t \to -\infty} 1 - e^{-\infty} = 1 - \frac{1}{e^{\infty}} = 1 - 0 = 1 = A_1$

$\int_{0}^{s} e^{-x}\,dx = \Big/_{0}^{s} -e^{-x} = -e^{-s} + e^{0} \xrightarrow{s \to \infty} -e^{-\infty} + 1 = 1 - \frac{1}{e^{\infty}} = 1 - 0 = 1 = A_2$

$\Rightarrow A = A_1 + A_2 = 1 + 1 = \underline{2}$

12.1 $f(x) = \begin{cases} 0, & x < 1 \\ \frac{a}{x^2}, & x \geq 1 \end{cases}$

a) 1° $f(x) \geq 0$: $\frac{a}{\underbrace{x^2}_{>0}} \geq 0 \Leftrightarrow a \geq 0$

2° $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{1} 0\,dx + \int_{1}^{\infty} \frac{a}{x^2}\,dx = \lim_{t \to \infty} \int_{1}^{t} \frac{a}{x^2}\,dx$

$= \lim_{t \to \infty} \int_{1}^{t} a x^{-2}\,dx = \lim_{t \to \infty} a \Big/_{1}^{t} \frac{1}{-1} x^{-1} = \lim_{t \to \infty} a\left(-\frac{1}{t} + \frac{1}{1}\right)$

$= a(0 + 1) = a = 1$

1° ja 2° $\Rightarrow \underline{a = 1}$

b) $P(X < 3) = \int_{-\infty}^{3} f(x)\,dx = \int_{-\infty}^{1} 0\,dx + \int_{1}^{3} \frac{1}{x^2}\,dx$

$= \int_{1}^{3} x^{-2}\,dx = \Big/_{1}^{3} \frac{1}{-1} x^{-1} = -\frac{1}{3} + \frac{1}{1} = \underline{\frac{2}{3} \approx 0{,}667}$

12.3 $f(x) = \begin{cases} 0, & x < 1 \\ \frac{3}{x^4}, & x \geq 1 \end{cases}$

a) $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{1} x \cdot 0\,dx + \int_{1}^{\infty} x \cdot \frac{3}{x^4}\,dx$

$= \lim_{t \to \infty} \int_{1}^{t} 3x^{-3}\,dx = \lim_{t \to \infty} \Big/_{1}^{t} 3 \cdot \frac{1}{-2} x^{-2}$

$= \lim_{t \to \infty} \left(-\frac{3}{2} \cdot \frac{1}{t^2} + \frac{3}{2} \cdot \frac{1}{1^2}\right) = 0 + \frac{3}{2} = \underline{\frac{3}{2}}$

b) $D(X) = \sqrt{\int_{-\infty}^{\infty} (x - E(X))^2 f(x)\,dx}$

$\int_{1}^{t} (x - E(X))^2 f(x)\,dx = \int_{1}^{t} \left(x - \frac{3}{2}\right)^2 \cdot \frac{3}{x^4}\,dx$

$= \int_{1}^{t} \left(x^2 - 3x + \frac{9}{4}\right) \cdot \frac{3}{x^4}\,dx = \int_{1}^{t} \left(\frac{3}{x^2} - \frac{9}{x^3} + \frac{27}{4x^4}\right) dx$

$= \Big/_{1}^{t} \left(3 \cdot \frac{1}{-1} x^{-1} - 9 \cdot \frac{1}{-2} x^{-2} + \frac{27}{4} \cdot \frac{1}{-3} x^{-3}\right)$

$= \left(-\frac{3}{t} + \frac{9}{2} \cdot \frac{1}{t^2} - \frac{9}{4} \cdot \frac{1}{t^3}\right) - \left(-\frac{3}{1} + \frac{9}{2} \cdot \frac{1}{1^2} - \frac{9}{4} \cdot \frac{1}{1^3}\right)$

$\xrightarrow{t \to \infty} (0 + 0 + 0) - \left(-3 + \frac{9}{2} - \frac{9}{4}\right) = \frac{3}{4}$

$\Rightarrow D(X) = \sqrt{\frac{3}{4}} = \underline{\frac{\sqrt{3}}{2}}$

12.5 X ; matkustajan odotusaika (min) (0...20)

a) X on luultavasti tasaisesti jakautunut välille 0...20. Jotta pinta-ala = 1, on vakioarvo $\frac{1}{2}$

$\Rightarrow f(x) = \begin{cases} \frac{1}{20}, & 0 \leq x \leq 20 \\ 0, & \text{muualla} \end{cases}$

b) $P(X > 15) = \int_{15}^{\infty} f(x)\,dx = \int_{15}^{20} \frac{1}{20}\,dx + \int_{20}^{\infty} 0\,dx = \Big/_{15}^{20} \frac{1}{20} x$

$= \frac{1}{20} \cdot 20 - \frac{1}{20} \cdot 15 = \frac{1}{4} = \underline{0{,}25}$

c) $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{20} x \cdot \frac{1}{20}\,dx = \frac{1}{20} \Big/_{0}^{20} \frac{1}{2} x^2$

$= \frac{1}{20} \cdot \left(\frac{1}{2} \cdot 20^2 - \frac{1}{2} \cdot 0^2\right) = \underline{10 \text{ (min)}}$

12.14 a) 2 (kuvaajan alle jäävä pinta-ala = 1)

b) 3 ($P(X \leq 2) = \frac{1}{2} \cdot 2 \cdot 0{,}25 = 0{,}25$)

c) 3 ($E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{0}^{4} x \cdot \frac{1}{8} x\,dx = \int_{0}^{4} \frac{1}{8} x^2\,dx$)

12.17 $f(x) = \begin{cases} 0, & x < 1 \\ \frac{1}{x^2}, & x \geq 1 \end{cases}$

$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{1}^{\infty} x \cdot \frac{1}{x^2}\,dx = \lim_{t \to \infty} \int_{1}^{t} \frac{1}{x}\,dx$

$= \lim_{t \to \infty} \Big/_{1}^{t} \ln x = \lim_{t \to \infty} (\ln t - \underbrace{\ln 1}_{=0}) = \ln \infty = \infty$

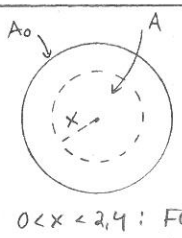
$\Rightarrow$ ei odotusarvoa

13.1 $F(x) = \begin{cases} 0, & x < 0 \\ 2^{\frac{1}{3}x} - 1, & 0 \leq x \leq 3 \\ 1, & x > 3 \end{cases}$

a) $P(X \leq 2) = F(2) = 2^{\frac{1}{3} \cdot 2} - 1 \approx \underline{0{,}587}$

b) $P(X > 1) = 1 - P(X \leq 1) = 1 - F(1) = 1 - (2^{\frac{1}{3} \cdot 1} - 1) \approx \underline{0{,}740}$

c) $P(1 < X \leq 4) = P(X \leq 4) - P(X \leq 1) = F(4) - F(1)$
$= 1 - (2^{\frac{1}{3} \cdot 1} - 1) \approx \underline{0{,}740}$

13.4 

X : nupin etäisyys keskipisteestä (0 ... 1,2), $r = \frac{2{,}4\,m}{2} = 1{,}2\,m$

a) $X \leq 0$: $F(x) = P(X \leq x) = 0$

$X \geq 2{,}4$: $F(x) = P(X \leq x) = 1$

$0 < x < 2{,}4$: $F(x) = P(X \leq x) \overset{\text{geom. tn}}{=} \frac{A}{A_0} = k^2 = \left(\frac{x}{1{,}2}\right)^2 = \frac{x^2}{1{,}44}$

$\Rightarrow F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{x^2}{1{,}44}, & 0 < x < 1{,}2 \\ 1, & x \geq 1{,}2 \end{cases}$

b) P(nuppi alle 25 cm reunasta) $= P(X \geq 0{,}95)$
$= 1 - P(X \leq 0{,}95) = 1 - F(0{,}95) = 1 - \frac{0{,}95^2}{1{,}44} \approx \underline{0{,}373}$