$$\lim_{x\to\infty} f(x) = x^4\left(1 - \frac{5}{x} + \frac{1}{x^2} + \frac{21}{x^3} - \frac{18}{x^4}\right) = \infty\,(1 - 0 + 0 + 0 - 0) = \infty\cdot 1 = \infty$$

$\Rightarrow$ ei suurinta arvoa

9.3

a) $\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{x+1}{x-2} = \frac{-2+1}{-2-2} = \frac{-1}{-4} = \frac{1}{4}$ $\Rightarrow$ on raja-arvo

b) $\lim_{x\to 2-} f(x) = \lim_{x\to 2-} \frac{\overbrace{x+1}^{>0}}{\underbrace{x-2}_{<0}} \overset{(\frac{3}{0})}{=} -\infty$

$\lim_{x\to 2+} f(x) = \lim_{x\to 2+} \frac{\overbrace{x+1}^{>0}}{\underbrace{x-2}_{>0}} \overset{(\frac{3}{0})}{=} \infty$

$\Rightarrow$ ei raja-arvoa eikä epäoleellista raja-arvoa

9.4

a) $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{1-x}{(x+3)^2} = \frac{1-1}{(1+3)^2} = \frac{0}{16} = 0$, on raja-arvo

b) $\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{1-x}{\underbrace{(x+3)^2}_{>0}} \overset{(\frac{4}{0})}{=} \infty$ $\Rightarrow$ ei raja-arvoa, epäoleellinen raja-arvo ∞

9.5

$f(x) = \frac{x^2}{x+1}$, f jatk. ja deriv. väl. $[-5,-1[$ ja $]-1,3]$

$f'(x) = \frac{2x(x+1) - x^2\cdot 1}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2} = 0 \quad |\cdot(x+1)^2 \neq 0$

$\Leftrightarrow x^2 + 2x = 0 \Leftrightarrow x(x+2) = 0 \Leftrightarrow x = \begin{cases} 0 \\ -2 \end{cases}$

	-5		-2		-1		0		3
f'		$+$		$-$		$-$		$+$	
f		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$	

os. -2, 0 min, >0

$f(-5) = -\frac{25}{4}$, $f(-2) = -4$, $f(0) = 0$, $f(3) = \frac{9}{4}$

$\lim_{x\to -1-} f(x) = \lim_{x\to -1-} \frac{x^2}{\underbrace{x+1}_{<0}} \overset{(\frac{1}{0})}{=} -\infty$

$\lim_{x\to -1+} f(x) = \lim_{x\to -1+} \frac{x^2}{\underbrace{x+1}_{>0}} \overset{(\frac{1}{0})}{=} \infty$

f saa kaikki arvot väleiltä $]-\infty,-4]$ ja $[0,\infty[$

9.7 a) 1 b) 3 c) 2

9.8

$f(x) = \frac{3x^2+4x}{\underbrace{x^2+1}_{>0}}$ f jatkuva ja derivoituva $\mathbb{R}$:ssä

$f'(x) = \frac{(6x+4)(x^2+1) - (3x^2+4x)\cdot 2x}{(x^2+1)^2} = \frac{-4x^2+6x+4}{(x^2+1)^2} = 0 \quad |\cdot(\,)^2$

$\Leftrightarrow -4x^2+6x+4 = 0 \Leftrightarrow x = \begin{cases} -\frac{1}{2} \\ 2 \end{cases}$

		$-\frac{1}{2}$		2	
f'	$-$		$+$		$-$
f	$\searrow$	min	$\nearrow$	max	$\searrow$

os. $-\frac{1}{2}$, 2 min, >0

$f(x) = \frac{3x^2+4x}{x^2+1} = \frac{x^2(3+\frac{4}{x})}{x^2(1+\frac{1}{x^2})} = \frac{3+\frac{4}{x}}{1+\frac{1}{x^2}} \xrightarrow{x\to\pm\infty} \frac{3+0}{1+0} = 3$

$f(-\frac{1}{2}) = -1 < \lim_{x\to\infty} f(x) = 3$ $\Rightarrow$ pienin arvo: -1

$f(2) = 4 > \lim_{x\to -\infty} f(x) = 3$ $\Rightarrow$ suurin arvo: 4

10.1

a) $\int_2^\infty \frac{4}{x^5}\,dx = I$

$\int_2^t \frac{4}{x^5}\,dx = \int_2^t 4x^{-5}\,dx = \Big/_2^t 4\cdot\frac{1}{-4}\overbrace{x^{-4}}^{\frac{1}{x^4}} = -\frac{1}{t^4} - \left(-\frac{1}{2^4}\right)$

$\xrightarrow{t\to\infty} -0 + \frac{1}{16} = \frac{1}{16} = I$

b) $\int_{-\infty}^{-3} \frac{5}{x}\,dx = I$

$\int_t^{-3} \frac{5}{x}\,dx = \Big/_t^{-3} 5\ln|x| = 5\ln|-3| - 5\ln|t| \xrightarrow{t\to-\infty} 5\ln 3 - 5\underbrace{\ln\infty}_{=\infty} = -\infty$ $\Rightarrow$ I hajaantuu

10.3

$A = \int_0^8 \frac{1}{\sqrt[3]{x}}\,dx$, ei määritelty kohdassa $x = 0$

$\int_t^8 \frac{1}{\sqrt[3]{x}}\,dx = \int_t^8 x^{-\frac{1}{3}}\,dx = \Big/_t^8 \frac{1}{\frac{2}{3}}x^{\frac{2}{3}} = \frac{3}{2}\cdot 8^{\frac{2}{3}} - \frac{3}{2}\cdot t^{\frac{2}{3}}$

$\xrightarrow{t\to 0+} \frac{3}{2}\cdot(\sqrt[3]{8})^2 - \frac{3}{2}\cdot(\sqrt[3]{0})^2 = \frac{3}{2}\cdot 2^2 - 0 = 6 = A$

10.5

a) $V = \pi\int_1^\infty (f(x))^2\,dx = \pi\int_1^\infty (3x^{-4})^2\,dx = \pi\int_1^\infty 9x^{-8}\,dx$

$\int_1^t 9x^{-8}\,dx = \Big/_1^t 9\cdot\frac{1}{-7}x^{-7} = -\frac{9}{7}\left(\frac{1}{t^7} - \frac{1}{1^7}\right)$

$\xrightarrow{t\to\infty} -\frac{9}{7}(0-1) = \frac{9}{7}$ $\Rightarrow$ $V = \frac{9}{7}\pi$

b) $V = \pi\int_1^\infty (f(x))^2\,dx = \pi\int_1^\infty (x^{-\frac{1}{4}})^2\,dx = \pi\int_1^\infty x^{-\frac{1}{2}}\,dx$

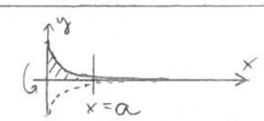
$\int_1^t x^{-\frac{1}{2}}\,dx = \Big/_1^t \frac{1}{\frac{1}{2}}x^{\frac{1}{2}} = 2\sqrt{t} - 2\sqrt{1}$

$\xrightarrow{t\to\infty} 2\cdot\infty - 2 = \infty$ $\Rightarrow$ tilavuudella ei ole arvoa

(tilavuus on ∞:n iso)

10.9

b) $V(a) = \pi\int_0^a (e^{-\pi x})^2\,dx$



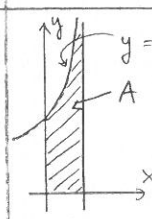
$= \pi\int_0^a e^{-2\pi x}\,dx$

$= \pi\cdot\frac{1}{-2\pi}\int_0^a -2\pi e^{-2\pi x}\,dx = -\frac{1}{2}\Big/_0^a e^{-2\pi x}$

$= -\frac{1}{2}\left[e^{-2\pi a} - e^0\right] \xrightarrow{a\to\infty} -\frac{1}{2}\left[e^{-\infty} - 1\right] = -\frac{1}{2}\left[\frac{1}{e^\infty} - 1\right]$

$= -\frac{1}{2}(0-1) = \frac{1}{2}$

10.13



$y = f(x) = \frac{2}{\sqrt{1-x}}$, f jatkuva välillä $[0,1[$

$\int_0^t \frac{2}{\sqrt{1-x}}\,dx = \int_0^t 2(1-x)^{-\frac{1}{2}}\,dx$

$= -2\int_0^t -(1-x)^{-\frac{1}{2}}\,dx = -2\Big/_0^t \frac{1}{\frac{1}{2}}(1-x)^{\frac{1}{2}}$

$= -4\left[\sqrt{1-t} - \sqrt{1-0}\right] \xrightarrow{t\to 1-} -4(\sqrt{0} - \sqrt{1}) = -4\cdot(-1) = 4 = A$

11.1

a) $\int_{-\infty}^\infty 2\,dx = \int_{-\infty}^0 2\,dx + \int_0^\infty 2\,dx = I_1 + I_2 = I$

$\int_t^0 2\,dx = \Big/_t^0 2x = 0 - 2t \xrightarrow{t\to-\infty} -2\cdot(-\infty) = \infty$

$\Rightarrow$ I_1 hajaantuu $\Rightarrow$ I hajaantuu

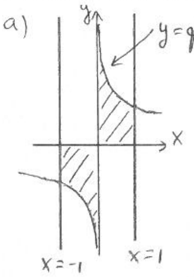
b) $\int_{-\infty}^\infty |x|\,dx = \int_{-\infty}^0 -x\,dx + \int_0^\infty x\,dx = I_1 + I_2 = I$

$\int_0^t x\,dx = \Big/_0^t \frac{1}{2}x^2 = \frac{1}{2}t^2 - 0 \xrightarrow{t\to\infty} \infty$

$\Rightarrow$ I_2 hajaantuu $\Rightarrow$ I hajaantuu

11.3

a)



$y = f(x) = \frac{1}{\sqrt[3]{x}}$, $x \neq 0$

$A = \int_{-1}^1 |f(x)|\,dx = \int_{-1}^1 \left|\frac{1}{\sqrt[3]{x}}\right|\,dx$

$= \int_{-1}^0 -\frac{1}{\sqrt[3]{x}}\,dx + \int_0^1 \frac{1}{\sqrt[3]{x}}\,dx = I_1 + I_2$

$\int_{-1}^t -\frac{1}{\sqrt[3]{x}}\,dx = -\int_{-1}^t x^{-\frac{1}{3}}\,dx = -\Big/_{-1}^t \frac{1}{\frac{2}{3}}x^{\frac{2}{3}}$

$= -\left[\frac{3}{2}t^{\frac{2}{3}} - \frac{3}{2}\cdot(-1)^{\frac{2}{3}}\right] = -\frac{3}{2}(\sqrt[3]{t})^2 + \frac{3}{2}(\sqrt[3]{-1})^2$

$\xrightarrow{t\to 0-} -\frac{3}{2}\cdot 0 + \frac{3}{2}\cdot 1 = \frac{3}{2} = I_1$

$\int_s^1 \frac{1}{\sqrt[3]{x}}\,dx = \ldots = \frac{3}{2}(\sqrt[3]{1})^2 - \frac{3}{2}(\sqrt[3]{s})^2 \xrightarrow{s\to 0+} \frac{3}{2}\cdot 1 - \frac{3}{2}\cdot 0 = \frac{3}{2} = I_2$

$\Rightarrow A = I_1 + I_2 = \frac{3}{2} + \frac{3}{2} = 3$