

Teil. $f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{x^2 g(x) - 0^2 g(0)}{x - 0}$

$$= \lim_{x \rightarrow 0} \frac{x^2 g(x)}{x} = \lim_{x \rightarrow 0} x g(x) = 0$$

$\downarrow$
 $0 \cdot -20 \leq \leq 16$

$\Rightarrow f$ an der Stelle 0 diff'bar (je $f'(0) = 0$)

3.6 $f(x) = 2\sqrt{x}$

$$f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{2\sqrt{x} - 2\sqrt{1}}{x - 1} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{x \rightarrow 1} \frac{2(\sqrt{x} - 1)}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{2(\sqrt{x} - 1)}{(\sqrt{x})^2 - 1^2} = \lim_{x \rightarrow 1} \frac{2(\sqrt{x} - 1)}{(\sqrt{x} - 1)(\sqrt{x} + 1)}$$

$$= \lim_{x \rightarrow 1} \frac{2}{\sqrt{x} + 1} = \frac{2}{\sqrt{1} + 1} = \frac{2}{2} = 1$$

Γ_{TA1} : $\lim_{x \rightarrow 1} \frac{\cancel{(\sqrt{x} - 1)} 2(\sqrt{x} - 1)}{x - 1} = \lim_{x \rightarrow 1} \frac{2 \overbrace{(\sqrt{x} - 1)(\sqrt{x} + 1)}^{= (\sqrt{x})^2 - 1^2 = \cancel{x - 1}}}{(\cancel{x - 1})(\sqrt{x} + 1)} = \dots$

3.10 Def. $f(x+y) = f(x) + f(y)$, $x, y \in \mathbb{R}$

$f'(0)$ an der Stelle

a) $f(0+0) = f(0) + f(0) \Rightarrow f(0) = 2f(0) \Leftrightarrow 0 = 2f(0) - f(0) \Rightarrow \underline{0 = f(0)}$

Γ_{TA1} : $f(x+0) = f(x) + f(0) \Rightarrow f(x) = f(x) + f(0) \quad | -f(x)$
 $\Rightarrow 0 = f(0)$

b) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{f(x) + \overbrace{f(h)}^{=0} - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{f(h)}{h} \stackrel{\left(\frac{0}{0}\right)}{=} \lim_{h \rightarrow 0} \frac{f(0+h) - \widetilde{f(0)}}{h} = f'(0)$$

$\Rightarrow$ warte

Γ $f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$

c) exm. $f(x) = x$