f on $\underline{x}$:n **tiheysfunktio** jos

$$\begin{cases} 1^\circ \; f(x) \geq 0 \text{ aina} \\ 2^\circ \; \int_{-\infty}^{\infty} f(x)\,dx = 1 \\ 3^\circ \; P(a \leq \underline{x} \leq b) = \int_a^b f(x)\,dx \end{cases}$$

$$\boxed{\begin{array}{l} E(\underline{x}) = \int_{-\infty}^{\infty} x f(x)\,dx \\ D(\underline{x}) = \sqrt{\int_{-\infty}^{\infty} (x - E(\underline{x}))^2 f(x)\,dx} \end{array}}$$

ODOTUSARVO

KESKIHAJONTA

12.4

$$f(x) = \begin{cases} 0 & , x \leq 0 \\ \frac{a}{\sqrt[4]{x}} & , 0 < x \leq 1 \\ 0 & , x > 1 \end{cases}$$

a) 1° $f(x) \geq 0$ aina: $\underbrace{\frac{a}{\sqrt[4]{x}}}_{>0} \geq 0 \iff a \geq 0$

2° $\int_{-\infty}^{\infty} f(x)\,dx = 1 \iff \int_{-\infty}^{0} 0\,dx + \int_0^1 \frac{a}{\sqrt[4]{x}}\,dx + \int_1^{\infty} 0\,dx$

$= \int_0^1 \frac{a}{\underbrace{\sqrt[4]{x}}_{x^{\frac{1}{4}}}}\,dx = \lim_{t \to 0+} a \int_t^1 x^{-\frac{1}{4}}\,dx = \lim_{t \to 0+} a \Big/_t^1 \frac{1}{\frac{3}{4}} x^{\frac{3}{4}}$

$= \lim_{t \to 0+} a\left(\frac{4}{3} \cdot 1^{\frac{3}{4}} - \frac{4}{3} \cdot \underbrace{t^{\frac{3}{4}}}_{\to 0}\right) = a \cdot \frac{4}{3} = 1 \quad \Big| \cdot \frac{3}{4}$

$\iff a = \frac{3}{4}$

1° ja 2° $\Rightarrow$ $\underline{\underline{a = \frac{3}{4}}}$

b) $E(\underline{x}) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 x \cdot \frac{\frac{3}{4}}{\sqrt[4]{x}}\,dx = \frac{3}{4} \int_0^1 x^1 \cdot x^{-\frac{1}{4}}\,dx$

$= \frac{3}{4} \int_0^1 x^{\frac{3}{4}}\,dx = \frac{3}{4} \Big/_0^1 \frac{4}{7} x^{\frac{7}{4}} = \frac{3}{4}\left(\frac{4}{7} \cdot 1^{\frac{7}{4}} - \frac{4}{7} \cdot 0^{\frac{7}{4}}\right)$

$= \frac{3}{4} \cdot \frac{4}{7} = \underline{\underline{\frac{3}{7}}}$