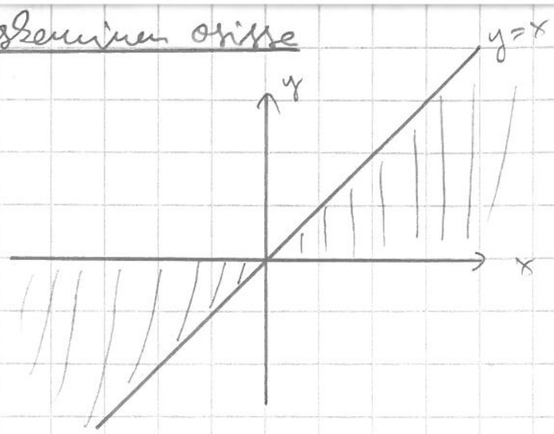
11. Epäoleellisen integraalin laskeminen osissa



Esim. Laske $\displaystyle\int_{-\infty}^{\infty} x\,dx = \int_{-\infty}^{0} x\,dx + \int_{0}^{\infty} x\,dx$

$$\int_t^0 x\,dx = \Big/_{t}^{0} \frac{1}{2}x^2 = \frac{1}{2}\cdot 0^2 - \frac{1}{2}t^2$$

$$\xrightarrow{t\to-\infty} 0 - \frac{1}{2}\cdot\infty = -\infty$$

$\Rightarrow \displaystyle\int_{-\infty}^{0} x\,dx$ hajaantuu $\Rightarrow \displaystyle\int_{-\infty}^{\infty} x\,dx$ hajaantuu

Yleisesti: 1° $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_{a}^{\infty} f(x)\,dx$

$$= \lim_{t\to-\infty}\int_t^a f(x)\,dx + \lim_{s\to\infty}\int_a^s f(x)\,dx \quad , a\in\mathbb{R}$$

2° $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$$= \lim_{t\to a+}\int_t^c f(x)\,dx + \lim_{s\to b-}\int_c^s f(x)\,dx \quad , t\in\,]a,b[$$

$f(a)$ ja $f(b)$ ei määritelty

3° $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$$= \lim_{t\to c-}\int_a^t f(x)\,dx + \lim_{t\to c+}\int_s^b f(x)\,dx \quad , f(c) \text{ ei ole määritelty}, c\in\,]a,b[$$

Epäoleellinen integraali suppenee $\Leftrightarrow$ molemmat raja-arvot ovat olemassa. Muussa tapauksessa integraali hajaantuu.

11.5 a) $\displaystyle I = \underbrace{\int_{-\infty}^{\infty} \frac{4x}{(x^2+1)^2}}_{\text{jatkuva } \mathbb{R}\text{:ssä}}\,dx = \underbrace{\int_{-\infty}^{0} \frac{4x}{(x^2+1)^2}\,dx}_{I_1} + \underbrace{\int_{0}^{\infty} \frac{4x}{(x^2+1)^2}\,dx}_{I_2}$

$$I_1: \int_t^0 \frac{4x}{(x^2+1)^2}\,dx = \int_t^0 4x(x^2+1)^{-2}\,dx = 4\cdot\frac{1}{2}\int_t^0 \underbrace{2x}_{f'(x)}\,\underbrace{(x^2+1)}_{f(x)}{}^{-2}\,dx$$

$$= 2\Big/_{t}^{0} \frac{1}{-1}(x^2+1)^{-1} = 2\left[-\frac{1}{0^2+1} + \underbrace{\frac{1}{t^2+1}}_{\to 0}\right] \xrightarrow{t\to-\infty} 2(-1+0) = -2$$

$$I_2: \int_0^s \frac{4x}{(x^2+1)^2}\,dx = 2\Big/_{0}^{s} \frac{1}{-1}(x^2+1)^{-1} = 2\left[-\underbrace{\frac{1}{s^2+1}}_{\to 0} + \frac{1}{0^2+1}\right] = 2(0+1) = 2$$