9. Funktion kulun tutkiminen

9.11 $f(x) = -12x^4 + 4x^3 + 9x^2 + \frac{3}{64}$

f jatk. ja derivoi. $R: \mathbb{R}$

$f'(x) = -48x^3 + 12x^2 + 18x = 0 \iff 6x(-8x^2 + 2x + 3) = 0$

$\iff x = 0$ tai $-8x^2 + 2x + 3 = 0$

$\iff x = \begin{cases} -\frac{1}{2} \\ \frac{3}{4} \end{cases}$

	+		−		+		−
f'		$-\frac{1}{2}$		0		$\frac{3}{4}$	
f	↗	max	↘	min	↗	max	↘

$f'(-1) = -48 \cdot (-1)^3 + 12 \cdot (-1)^2 + 18 \cdot (-1) = 42 > 0$

$f'(-0{,}1) = \ldots \approx -1{,}6 < 0$

$f'(\frac{1}{2}) = 6 > 0$

$f(1) = -18 < 0$

$\begin{cases} f(-\frac{1}{2}) = -12 \cdot (-\frac{1}{2})^4 + 4 \cdot (-\frac{1}{2})^3 + 9 \cdot (-\frac{1}{2})^2 + \frac{3}{64} = \frac{67}{64} \\ f(\frac{3}{4}) = \ldots = 3 \end{cases}$ $\Rightarrow$ suurin arvo: 3

$f(0) = \frac{3}{64}$

$f(x) = -12x^4 + 4x^3 + 9x^2 + \frac{3}{64} = x^4\left(-12 + \frac{4}{x} + \frac{9}{x^2} + \frac{\frac{3}{64}}{x^4}\right)$

$\xrightarrow{x \to \pm\infty} \infty \cdot (-12 + 0 + 0 + 0) = -\infty$

$\Rightarrow$ ei ole pienintä arvoa

a) f jatkuva $\Rightarrow$ arvojoukko $]-\infty, 3] = A_f$

9.14 $f(x) = \frac{3x - x^2}{x - 4}$, $x \neq 4$

$D\frac{f}{g} = \frac{f'g - fg'}{g^2}$

f jatk. ja derivoi. kun $x \neq 4$

$f'(x) = \frac{(3 - 2x)(x - 4) - (3x - x^2) \cdot 1}{(x - 4)^2} = \frac{3x - 12 - 2x^2 + 8x - 3x + x^2}{(x - 4)^2}$

$= \frac{-x^2 + 8x - 12}{(x - 4)^2} = 0 \quad | \cdot (x - 4)^2$

$\iff -x^2 + 8x - 12 = 0 \iff x = \begin{cases} 2 \\ 6 \end{cases}$

	−		+		+		−
f'		2		4		6	
f	↘	min	↗		↗	max	↘

$f'(0) = -\frac{3}{4} < 0$ $\quad f'(3) = 3 > 0$

$f'(5) = 3 > 0$ $\quad f'(7) = -\frac{5}{9} < 0$