

$$\begin{aligned}
 6) \lim_{x \rightarrow -\infty} \frac{6x+1}{\sqrt{9x^2+7}} &= \lim_{x \rightarrow -\infty} \frac{6x+1}{\sqrt{x^2(9+\frac{7}{x^2})}} = \lim_{x \rightarrow -\infty} \frac{6x+1}{\underbrace{\sqrt{x^2}}_{|x|=-x} \sqrt{9+\frac{7}{x^2}}} \\
 &= \lim_{x \rightarrow -\infty} \frac{x(6+\frac{1}{x})}{-x \sqrt{9+\frac{7}{x^2}}} = \lim_{x \rightarrow -\infty} \frac{6+\frac{1}{x}}{-\sqrt{9+\frac{7}{x^2}}} = \frac{6+0}{-\sqrt{9+0}} = \frac{6}{-3} = -2
 \end{aligned}$$

$$\begin{aligned}
 & \lceil (\sqrt{x})^2 = x, x \geq 0 \\
 & \sqrt{x^2} = |x|, x \in \mathbb{R} \rceil
 \end{aligned}$$

$$8.6 \text{ a) } \lim_{x \rightarrow \infty} \frac{1}{7^x} = \frac{1}{7^\infty} = \frac{1}{\infty} = 0$$

$$\begin{aligned}
 \text{b) } \lim_{x \rightarrow -\infty} \left(\frac{7}{3}\right)^{2x} &= \lim_{y \rightarrow \infty} \left(\frac{7}{3}\right)^{-2y} = \lim_{y \rightarrow \infty} \frac{1}{\left(\frac{7}{3}\right)^{2y}} = \frac{1}{\infty} = 0
 \end{aligned}$$

$x = -y$
 $-1 \leq x \leq 1$
 $y > 1$

$$8.19 \text{ a) } f(x) = \frac{\sin 6x}{x+2} \xrightarrow{x \rightarrow \infty} 0$$

$-1 \leq \sin x \leq 1$
 $x \rightarrow \infty$

Osoitetaan on arvo luku jolla on välillä $[-9, 9]$ ja määrittäjä $\rightarrow \infty$. Siten $f(x)$ lähestyy lukua 0 kun $x \rightarrow \infty$

$$\begin{aligned}
 \text{b) } f(x) &= x e^{\sin x} \xrightarrow{x \rightarrow \infty} \infty \\
 &\quad \downarrow \quad \downarrow \\
 &\quad \infty \quad e^{-1} \leq e^1 \leq e^1 \\
 &\quad \quad \approx 0,37 \quad \approx 2,7
 \end{aligned}$$

Koska x kasvaa rajatta ($x \rightarrow \infty$) ja $e^{\sin x}$ on arvo välillä $[\frac{1}{e}, e]$

$\Rightarrow$ epäoleellinen rajo-arvo ∞

8.20

$$\boxed{\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e}$$

$$\text{a) } \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{-x} = \lim_{x \rightarrow \infty} \frac{1}{\left(1 + \frac{1}{x}\right)^x} = \frac{1}{e}$$

$$\text{b) } \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{2x} = \lim_{x \rightarrow \infty} \left(\left(1 + \frac{1}{x}\right)^x\right)^2 = e^2$$

$$\text{c) } \lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^x = \lim_{x \rightarrow \infty} \underbrace{\left(1 + \frac{1}{2x}\right)^{2x}}_{\rightarrow e}^{\frac{1}{2}} = e^{\frac{1}{2}} = \sqrt{e}$$