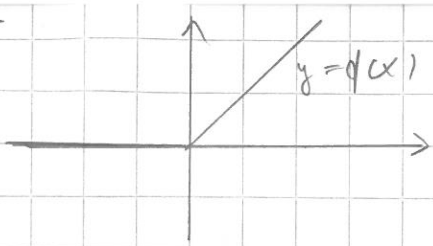
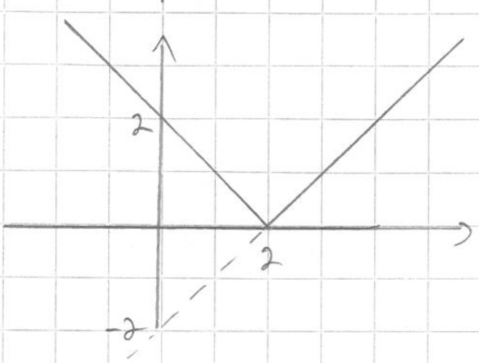
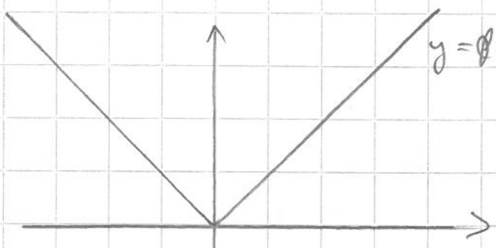
4.15



$$f(x) = \begin{cases} 0 & , x \le 0 \\ x & , x > 0 \end{cases}$$



$$f(x) = \begin{cases} -x+2 & , x < 2 \\ x-2 & , x \ge 2 \end{cases}$$



$$y = f(x) = \begin{cases} -x & , x \le 0 \\ x & , x \ge 0 \end{cases} \qquad (= |x|)$$

f on jatkuva ja derivoituva ainakin kun $x \neq 0$

1° $$\begin{cases} \lim\limits_{x \to 0-} f(x) = \lim\limits_{x \to 0-} (-x) = 0 \\ \lim\limits_{x \to 0+} f(x) = \lim\limits_{x \to 0+} x = 0 \\ f(0) = 0 \end{cases}$$

$\Rightarrow \lim\limits_{x \to 0-} f(x) = \lim\limits_{x \to 0+} f(x) = f(0)$

$\Rightarrow f$ on jatkuva 0:ssa

$\Rightarrow f$ on jatkuva $\mathbb{R}$:ssä

2° $$\begin{cases} f'_-(0) = \lim\limits_{x \to 0-} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0-} \dfrac{-x - 0}{x} = \lim\limits_{x \to 0-} (-1) = -1 \\ f'_+(0) = \lim\limits_{x \to 0+} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0+} \dfrac{x - 0}{x - 0} = \lim\limits_{x \to 0+} 1 = 1 \end{cases}$$

$\Rightarrow f'_-(0) \neq f'_+(0) \Rightarrow f$ ei ole derivoituva kohdassa 0

4.19 a) $$f(x) = \begin{cases} x \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

$$f'(0): \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\cancel{x} \sin \frac{1}{x} - 0}{\cancel{x}} = \lim_{x \to 0} \sin \underbrace{\frac{1}{x}}_{\to \infty}$$ ei raja-arvoa

sinin arvot heiluvat välillä $[-1, 1]$

$\Rightarrow f'(0)$ ei ole olemassa $\Rightarrow$ ei ole

b) $$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$