3.6 $f(x) = 2\sqrt{x}$

$$f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{2\sqrt{x}-2\cdot\sqrt{1}}{x-1} = \lim_{x\to 1}\frac{2\sqrt{x}-2}{x-1} \left(\frac{0}{0}\right) = \lim_{x\to 1} \frac{2(\sqrt{x}-1)}{\underset{(\sqrt{x})^2}{x} - \underset{1^2}{1}}$$

$$= \lim_{x\to 1} \frac{2\cancel{(\sqrt{x}-1)}}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)} = \lim_{x\to 1}\frac{2}{\sqrt{x}+1} = \frac{2}{\sqrt{1}+1} = \frac{2}{2} = \underline{1}$$

$$\left\ulcorner \frac{2(\sqrt{x}-1)}{x-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{\overbrace{2(\sqrt{x}-1)(\sqrt{x}+1)}^{(\sqrt{x})^2-1^2}}{(x-1)(\sqrt{x}+1)} = \frac{2\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)} = \frac{2}{\sqrt{x}+1} \right\lrcorner$$

3.10 Olet. $f(x+y) = f(x) + f(y)$, $x, y \in \mathbb{R}$

$f'(0)$ on olemassa

a) Olet. $\Rightarrow f(x+0) = f(x) + f(0) \Leftrightarrow f(x) = f(x) + f(0) \Leftrightarrow \underline{f(0) = 0}$

b) $$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \overset{\text{olet.}}{=} \lim_{h\to 0}\frac{f(x)+f(h)-f(x)}{h} = \lim_{h\to 0}\frac{f(h)}{h}$$

$$= \lim_{h\to 0}\frac{f(0+h) - \overbrace{f(0)}^{=0}}{h} = f'(0), \quad x\in\mathbb{R} \quad \Rightarrow \text{väite}$$

$$\ulcorner f'(0) = \lim_{h\to 0}\frac{f(0+h)-\overbrace{f(0)}^{=0}}{h} = \lim_{h\to 0}\frac{f(h)}{h} \lrcorner$$

c) $f'(x) = f'(0) \overset{\text{merk.}}{=} a \in \mathbb{R}$

$$\Rightarrow \underline{f(x)} = \int a\,dx = \underline{ax + C \quad , a, C \in \mathbb{R}}$$

Esim. $f(x) = x$

3.11