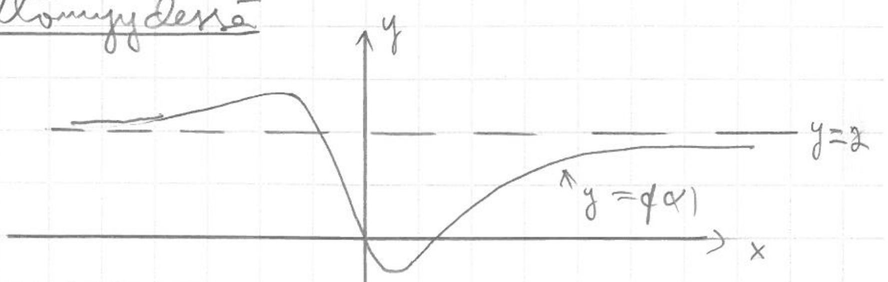
# 8. Raja-arvo äärettömyydessä

Esim. $f(x) = \dfrac{2x^2-3x}{x^2+1}$




$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{2x^2-3x}{x^2+1} \overset{\left(\frac{\infty-\infty}{\infty}\right)}{=} \lim_{x\to\pm\infty} \frac{\cancel{x^2}\left(2-\frac{3}{x}\right)}{\cancel{x^2}\left(1+\frac{1}{x^2}\right)} = \lim_{x\to\pm\infty} \frac{2-\overbrace{\frac{3}{x}}^{\to 0}}{1+\underbrace{\frac{1}{x^2}}_{\to 0}} = \frac{2-0}{1+0} = \underline{2}$$

Yleisesti: $\lim\limits_{x\to\pm\infty} \dfrac{f(x)}{g(x)}$, $f$ ja $g$ polynomeja

1° $g$:n korkein potenssi yhteiseksi tekijäksi $f$:stä ja $g$:stä
2° supistetaan
3° lasketaan raja-arvo ( $\lim\limits_{x\to\pm\infty} \dfrac{1}{x^n} = 0$, $n = 1, 2, 3, \ldots$ )

Epämääräisiä muotoja: $\dfrac{\infty}{\infty}$, $\infty-\infty$, $0\cdot\infty$

8.2 a) $$\lim_{x\to\infty} \frac{8x^2-3x}{4x^2-5} \overset{\left(\frac{\infty-\infty}{\infty}\right)}{=} \lim_{x\to\infty} \frac{\cancel{x^2}\left(8-\frac{3}{x}\right)}{\cancel{x^2}\left(4-\frac{5}{x^2}\right)} = \lim_{x\to\infty} \frac{8-\overbrace{\frac{3}{x}}^{\to 0}}{4-\underbrace{\frac{5}{x^2}}_{\to 0}} = \frac{8-0}{4-0} = \underline{2}$$

b) $$\lim_{x\to-\infty} \frac{5x^2+2x}{x^2} = \lim_{x\to-\infty} \frac{\cancel{x^2}\left(5+\frac{2}{x}\right)}{\cancel{x^2}} = \lim_{x\to-\infty} \left(5+\underbrace{\frac{2}{x}}_{\to 0}\right) = 5+0 = \underline{5}$$

– $\left(-\infty-(-\infty)^5 = -\infty-(-\infty) = -\infty+\infty\right)$

8.4 a) $$\lim_{x\to-\infty} (9x-x^5) = \lim_{x\to-\infty} x^5\left(\frac{9}{x^4}-1\right) = (-\infty)^5\left(\frac{9}{(-\infty)^4}-1\right)$$
$$= -\infty\cdot\left(\frac{9}{\infty}-1\right) = -\infty\cdot(0-1) = \infty$$

ei raja-arvoa, epäoleellinen raja-arvo $\infty$

b) $$\lim_{x\to-\infty} \frac{6x+1}{\sqrt{9x^2+7}} \overset{\left(\frac{-\infty}{\infty}\right)}{=} \lim_{x\to-\infty} \frac{6x+1}{\sqrt{x^2\left(9+\frac{7}{x^2}\right)}} = \lim_{x\to-\infty} \frac{6x+1}{\sqrt{x^2}\sqrt{9+\frac{7}{x^2}}}$$
$$= \lim_{x\to-\infty} \frac{x\left(6+\frac{1}{x}\right)}{\underbrace{|x|}_{<0}\sqrt{9+\frac{7}{x^2}}} = \lim_{x\to-\infty} \frac{\cancel{x}\left(6+\frac{1}{x}\right)}{-\cancel{x}\sqrt{9+\frac{7}{x^2}}}$$
$$= \lim_{x\to-\infty} \frac{6+\frac{1}{x}}{-\sqrt{9+\frac{7}{x^2}}} = \frac{6+0}{-\sqrt{9+0}} = \frac{6}{-3} = \underline{-2}$$