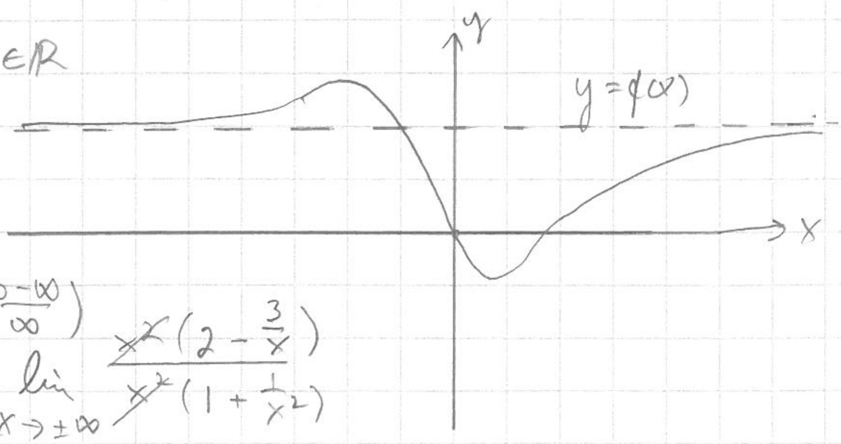
8. Raja-arvo äärettömyydessä

Esim. $f(x) = \dfrac{2x^2-3x}{\underbrace{x^2+1}_{\geq 0}}$, $x \in \mathbb{R}$



$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{2x^2-3x}{x^2+1}\left(\frac{\infty-\infty}{\infty}\right) = \lim_{x\to\pm\infty} \frac{\cancel{x^2}\left(2-\frac{3}{x}\right)}{\cancel{x^2}\left(1+\frac{1}{x^2}\right)}$$

$$= \lim_{x\to\pm\infty} \frac{2-\frac{3}{x}\to 0}{1+\underbrace{\frac{1}{x^2}}_{\to 0}} = \frac{2-0}{1+0} = \underline{2}$$

Yleisesti $\lim\limits_{x\to\pm\infty} \dfrac{f(x)}{g(x)}$; f ja g polynomeja

1° g:n korkein potenssi yhteiseksi tekijäksi f:stä ja g:stä
2° supistetaan
3° lasketaan raja-arvo ($\lim\limits_{x\to\pm\infty} \dfrac{a}{x^n} = 0$, $n = 1,2,\ldots$, $a \in \mathbb{R}$ (vakio))

Epämääräisiä muotoja ($\frac{\infty}{\infty}$, $\infty-\infty$, $0\cdot\infty$)

8.2 a) $\lim\limits_{x\to\infty} \dfrac{8x^2-3x}{4x^2-5}\left(\dfrac{\infty-\infty}{\infty}\right) = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}\left(8-\frac{3}{x}\right)}{\cancel{x^2}\left(4-\frac{5}{x^2}\right)} = \lim\limits_{x\to\infty} \dfrac{8-\frac{3}{x}\to 0}{4-\frac{5}{x^2}\to 0} = \dfrac{8-0}{4-0} = \dfrac{8}{4} = \underline{2}$

b) $\lim\limits_{x\to-\infty} \dfrac{5x^2+2x}{x^2} = \lim\limits_{x\to-\infty} \dfrac{\cancel{x^2}\left(5+\frac{2}{x}\right)}{\cancel{x^2}} = \lim\limits_{x\to-\infty} 5+\dfrac{2}{x} = 5+0 = \underline{5}$

$(-\infty-(-\infty) = -\infty+\infty)$

8.4 a) $\lim\limits_{x\to-\infty} (9x-x^5) = \lim\limits_{x\to-\infty} x^5\left(\dfrac{9}{x^4}-1\right) = (-\infty)^5\left(\dfrac{9}{(-\infty)^4}-1\right)$

$= -\infty\cdot(0-1) = \infty$

b) $\lim\limits_{x\to-\infty} \dfrac{6x+1}{\sqrt{9x^2+7}}\left(\dfrac{-\infty}{\infty}\right) = \lim\limits_{x\to-\infty} \dfrac{6x+1}{\sqrt{x^2\cdot\left(9+\frac{7}{x^2}\right)}} = \lim\limits_{x\to-\infty} \dfrac{6x+1}{\underbrace{\sqrt{x^2}}_{|x|=-x}\sqrt{9+\frac{7}{x^2}}}$

$|x| = -x$ ← $x<0$ koska $x\to-\infty$

$= \lim\limits_{x\to-\infty} \dfrac{\cancel{x}\left(6+\frac{1}{x}\right)}{-\cancel{x}\sqrt{9+\frac{7}{x^2}}} = \lim\limits_{x\to-\infty} \dfrac{6+\frac{1}{x}\to 0}{-\sqrt{9+\frac{7}{x^2}}\to 0} = \dfrac{6+0}{-\sqrt{9+0}} = \dfrac{6}{-3} = \underline{-2}$

Yleisesti f polynomi

$\lim\limits_{x\to\pm\infty} f(x) = \infty$ tai $-\infty$, lopputuloksen (∞ tai $-\infty$) määrää