

$$f(x) = x^3 + x + 3 = 1 \quad (\Leftrightarrow) x = -1$$

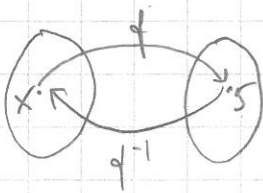
$$(f^{-1})'(1) = \frac{1}{f'(-1)} = \frac{1}{3 \cdot (-1)^2 + 1} = \frac{1}{4}$$

$$f(x) = x^3 + x + 3 = -7 \quad (\Leftrightarrow) x = -2$$

$$(f^{-1})'(-7) = \frac{1}{f'(-2)} = \frac{1}{3 \cdot (-2)^2 + 1} = \frac{1}{13}$$

7.17 $f(x) = \ln x + 2x + 3$

$f(x) = \ln x + 2x + 3 = y \quad (\Leftrightarrow) x = \dots = f^{-1}(y)$



$$f(x) = \ln x + 2x + 3 = 5 \quad (\Leftrightarrow) x = 1$$

$$f'(x) = \frac{1}{x} + 2$$

$$k_t = (f^{-1})'(5) = \frac{1}{f'(1)} = \frac{1}{\frac{1}{1} + 2} = \frac{1}{3}$$

punkte: (5, 1)

tangentl.: $y - 1 = \frac{1}{3}(x - 5) \quad (\Leftrightarrow) y = \frac{1}{3}x - \frac{2}{3}$

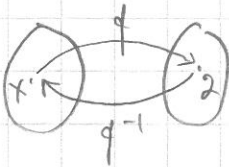
7.19 $f(x) = \ln x + x + 1, x > 0$

a) f jelt. jö deriv. kun $x > 0$

$$f'(x) = \frac{1}{x} + 1 > 0 \text{ aino kun } x > 0 \Rightarrow f \text{ aidosti kasvava}$$

\Rightarrow käänteisfunktio $f^{-1} = g$ on olemassa

b)



$$f(x) = \ln x + x + 1 = 2 \quad (\Leftrightarrow) x = 1$$

$$g'(2) = (f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{\frac{1}{1} + 1} = \frac{1}{2}$$

c) f :n ja f^{-1} :n kuvaajat ovat toistensa peilikuvia suoran $y = x$ suhteen \Rightarrow niiden leikkauspisteet ovat suoralla $y = x$

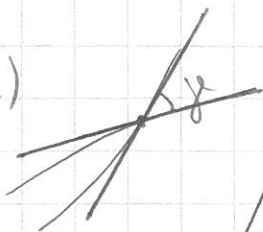
$$\begin{cases} y = f(x) = \ln x + x + 1 \\ y = x \end{cases}$$

$$\Rightarrow \ln x + x + 1 = x \quad (\Leftrightarrow) \ln x = -1 \quad | e^{(\cdot)}$$

$$\Rightarrow e^{\ln x} = e^{-1} \quad (\Leftrightarrow) x = e^{-1} = \frac{1}{e}$$

$$\Rightarrow \text{leikkauspiste: } \left(\frac{1}{e}, \frac{1}{e}\right)$$

d)



$$k_{t1} = f'\left(\frac{1}{e}\right) = \frac{1}{\frac{1}{e}} + 1 = e + 1 = \tan \alpha \Rightarrow \alpha = 74,95^\circ$$

$$k_{t2} = (f^{-1})'\left(\frac{1}{e}\right) = \frac{1}{f'\left(\frac{1}{e}\right)} = \frac{1}{e + 1} = \tan \beta = 15,05^\circ$$

$$\Rightarrow \gamma = \alpha - \beta = 59,9^\circ$$