$$\begin{cases} \lim\limits_{x\to 2+} f(x) = \lim\limits_{x\to 2+} (6x-4) = 6\cdot 2 - 4 = 8 \\ f(2) = a\cdot 2^2 + b = 4a + b \end{cases}$$

$\Rightarrow \quad 4a + b = 8$

2° $$f'_-(2) = \lim_{x\to 2-} \frac{f(x) - f(2)}{x-2} = \lim_{x\to 2-} \frac{(ax^2+b) - (4a+b)}{x-2} = \lim_{x\to 2-} \frac{ax^2 - 4a}{x-2}$$

$$\overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 2-} \frac{a(x^2 - 2^2)}{x-2} = \lim_{x\to 2-} \frac{a\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \lim_{x\to 2-} \left(a(x+2)\right) = a(2+2) = 4a$$

$$f'_+(2) = \lim_{x\to 2+} \frac{f(x) - f(2)}{x-2} = \lim_{x\to 2+} \frac{(6x-4) - \overbrace{(a\cdot 2^2 + b)}^{4a+b=8 \text{ (1° kohta)}}}{x-2} = \lim_{x\to 2+} \frac{6x-4-8}{x-2}$$

$$= \lim_{x\to 2+} \frac{6x-12}{x-2} = \lim_{x\to 2+} \frac{6\cancel{(x-2)}}{\cancel{x-2}} = \lim_{x\to 2+} 6 = 6$$

f derivoituva kohdassa 2 $\Leftrightarrow f'_-(2) = f'_+(2) \Leftrightarrow 4a = 6 \Leftrightarrow \underline{a = \frac{3}{2}}$

1° $\Rightarrow 4a + b = 8 \Leftrightarrow \underline{b} = 8 - 4a = 8 - 4\cdot\frac{3}{2} = \underline{2}$

4.14 $f: \mathbb{R} \to \mathbb{R}$ ← $f(x) \in \mathbb{R}$
↑
määrittelyjoukko $(x \in \mathbb{R})$

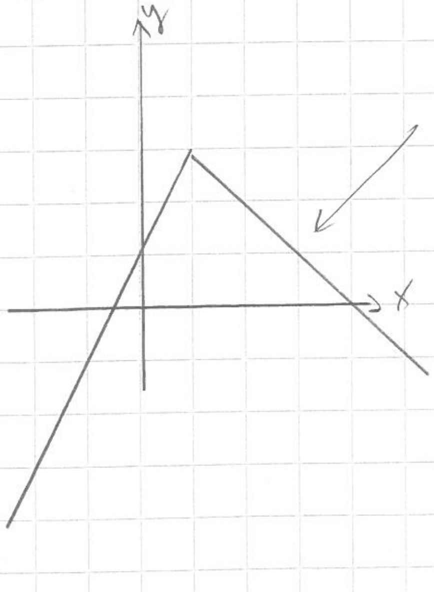
a) $f'_-(2) = 1$

b) $$f'_+(2): \lim_{x\to 2+} \frac{f(x) - f(2)}{x-2} = \lim_{x\to 2+} \frac{2-1}{x-2}$$

$$= \lim_{x\to 2+} \frac{1}{\underbrace{x-2}_{>0 \text{ aina}}}$$

Rajalla tilanne $\frac{1}{0} = \infty \Rightarrow$ raja-arvoa ei ole olemassa $\Rightarrow f'_+(2)$ ei ole olemassa

4.15



$$y = f(x) = \begin{cases} 2x+1, & x \le 1 \\ -x+4, & x > 1 \end{cases}$$

f jatk. ja deriv. ainakin kohdissa $x \ne 1$

1° $$\begin{cases} \lim\limits_{x\to 1-} f(x) = \lim\limits_{x\to 1-} (2x+1) = 2\cdot 1 + 1 = 3 \\ \lim\limits_{x\to 1+} f(x) = \lim\limits_{x\to 1+} (-x+4) = -1 + 4 = 3 \\ f(1) = 2\cdot 1 + 1 = 3 \end{cases}$$

Koska $\lim\limits_{x\to 1-} f(x) = \lim\limits_{x\to 1+} f(x) = f(1) \quad (= 3)$

$\Rightarrow$ f on jatkuva kohdassa 1

$\Rightarrow$ f on kaikkialla jatkuva