3.4 $f(x) = x^2+3x$

$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{(x^2+3x)-0}{x} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 0}\left(\frac{x^2}{x}+\frac{3x}{x}\right)$$

$$= \lim_{x\to 0}(x+3) = 0+3 = \underline{3}$$

$$\left\ulcorner\; f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{(h^2+3h)-0}{h} = \ldots = 3 \;\right\lrcorner$$

3.5 <u>Olet.</u> $-20 \le g(x) \le 16$, $x \in \mathbb{R}$

$f(x) = x^2 g(x)$

<u>Väite</u> f on derivoituva kohdassa $x=0$

<u>Tod.</u>
$$f'(0) = \lim_{x\to 0}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0}\frac{x^2g(x)-0^2g(0)}{x}$$

$$= \lim_{x\to 0}\frac{x^2g(x)}{x} = \lim_{x\to 0}(\underbrace{x}_{\to 0}\underbrace{g(x)}_{-20\le\;\le 16}) = 0$$

$\Rightarrow$ f on derivoituva kohdassa 0 m.o.t.

3.6 $f(x) = 2\sqrt{x}$

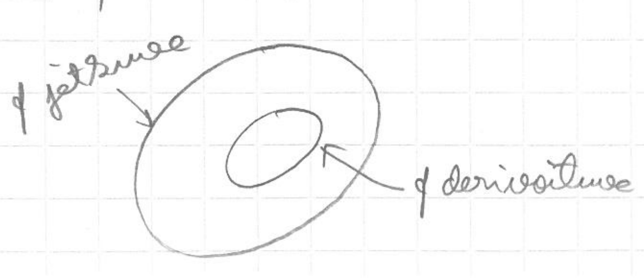
$$f'(1) = \lim_{x\to 1}\frac{f(x)-f(1)}{x-1} = \lim_{x\to 1}\frac{2\sqrt{x}-2}{x-1} \overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 1}\frac{2(\sqrt{x}-1)}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$= \lim_{x\to 1}\frac{2(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1}\frac{2\left((\sqrt{x})^2-1^2\right)}{(x-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1}\frac{2\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)} = \frac{2}{\sqrt{1}+1} = \frac{2}{2} = \underline{1}$$

4. <u>Toispuoliset derivaatat</u>

<u>Lause</u> f on derivoituva kohdassa a $\Rightarrow$ f on jatkuva kohdassa a



Siis: jos f ei ole jatkuva, se ei ole derivoituva