PROBLEMS (Raymond Smullyan: What is the name of this book?)

A. The Forest of Werewolves

Suppose you are visiting a forest in which every inhabitant is either a knight or a knave. Knights always tell the truth and knaves always lie. In addition, some of the inhabitants are werewolves and have the annoying habit of sometimes turning into wolves at night and devouring people. A werewolf can be either a knight or a knave.

1. You are interviewing three inhabitants, A, B and C, and it is known that exactly one of them is a werewolf. They make the following statements:

   A: C is a werewolf.
   B: I am not a werewolf.
   C: At least two of us are knaves.

   Our problem has two parts:
   a) Is the werewolf a knight or a knave?
   b) If you have to take one of them as a travelling companion, and it is more important that he not be a werewolf than that he not be a knave, which one would you pick?

2. Again, each of A, B and C is a knight or a knave and exactly one of them is a werewolf. They make the following statements:

   A: I am a werewolf.
   B: I am a knight.
   C: At most one of us is a knight.

   Give a complete classification of A, B and C.

3. In this and the next two problems there are again three inhabitants A, B and C, each of whom is either a knight or a knave. However only two of them, A and B, make statements. But in these statements the word 'us' refers to the three people A, B and C – not just A and B.

   Suppose A and B make the following statements:
   A: At least one of the three of us is a knight.
   B: At least one of the three of us is a knave.

   Given that at least one of them is a werewolf, and that none of them is both a knight and a werewolf, which ones are werewolves?

4. This time we get the following statements:
   A: At least one of the three of us is a knave.
   B: C is a knight.

   Given that there is exactly one werewolf and that he is a knight, who is the werewolf?

5. In this problem we get the following two statements:
   A: At least one of the three of us is a knave.
   B: C is a werewolf.

   Again, there is exactly one werewolf and he is a knight. Who is he?

6. In this problem we are given that there is exactly one werewolf and that he is a knight, and that the other two are knaves. Only one of them, B, makes a statement: "C is a werewolf". Who is the werewolf?

7. Here is an elegantly simple one involving just two inhabitants, A and B. Jus one of them is a werewolf. They make the following statements:
   A: The werewolf is a knight.
   B: The werewolf is a knave.

   Which one would you select for your travelling companion?

B. How to Win or Choose a Bride

8. Suppose you are an inhabitant of the island of knights and knaves. You fall in love with a girl there and wish to marry her. However, this girl has a strange tastes; for some odd reason she does not wish to marry a knight; she wants to marry only a knave. But she wants a rich knave, not a poor one. (We assume for convenience that everyone there is classified as either rich or poor.) Suppose, in fact, that you are a rich knave. You are allowed to make only one statement to her. How, in only one statement, can you convince her that you are a rich knave?

9. Suppose, instead, the girl you love wants to marry only a rich knight. How, in one statement, could you – a rich knight – convince her that you are a rich knight?

10. This time you are a visitor to the island of knights and knaves. Every female there is either a knight or a knave. You fall in love with one of the females there. – a girl named Elizabeth – and are thinking of marrying her. However, you want to know just what you are getting into; you do not wish to marry a knave. If you were allowed to question her, there would be no problem, but an ancient taboo of the island forbids a man to hold speech with any female unless he is already married to her. However, Elizabeth has a brother Arthur who is also a knight or a knave (but not necessarily the same as his sister). You are allowed to ask just one question of the the brother, but the question must be answerable by "Yes" or "No".

   The problem is for you to design a question such that upon hearing the answer, you will know for sure wheather Elizabeth is a knight or a knave. What question would you ask?

C. Yes, You Are Innocent, but Can You Prove It?

A crime has been committed on the island of knights and knaves, and for some strange reason it is suspected that you are the criminal. You are brought to court and tried. You are allowed to make only one statement on your own behalf. Your purpose is to convince the jury you are innocent.

11. Suppose it is known that the criminal is a knave. Suppose also that you are a knave (though the court doesn't know this) but that you are nevertheless innocent of this crime. How can you convince the jury that you are innocent of the crime?

12. Suppose you are in the same situation except for the fact that you are guilty. What statement could you make which would convince the jury (assuming they are rational beings) that you are innocent?

13. In this problem, suppose it is known that the criminal is a knight. (That is no contradiction; a person doesn't necessarily have to lie in order to commit a crime.) Suppose also that you are a knight (but the jury doesn't know this) but innocent of the crime. What statement would you make?

14. Deduction: "If there is God, He is good and almighty. If God is good and almighty no evil happens in the world. There happens evil in the world. Thus there is no God."
   a) Is the deduction valid? b) Has it been proved that there is no God?