4.16 <u>Väite</u> $n(n+1)(2n+1)$ on jaollinen luvulla 6 kun $n \in \mathbb{Z}$

<u>Tod.</u> n on jotakin seuraavista muodoista:

1° $n = 6k$: $n(n+1)(2n+1) = 6\underbrace{k(6k+1)(2 \cdot 6k+1)}_{\in \mathbb{Z}}$ // on jaollinen 6:lla

2° $n = 6k+1$: $n(n+1)(2n+1) = (6k+1)(6k+1+1)(2(6k+1)+1)$

$= (6k+1)(6k+2)(12k+3)$

$= 2(6k+1)(3k+1) \cdot 3(4k+1)$

$= 6\underbrace{(6k+1)(3k+1)(4k+1)}_{\in \mathbb{Z}}$ // on jaollinen 6:lla

3° $n = 6k+2$: $n(n+1)(2n+1) = (6k+2)(6k+2+1)(2(6k+2)+1)$

$= 2(3k+1)\,3(2k+1)(12k+5)$

$= 6\underbrace{(3k+1)(2k+1)(12+5)}_{\in \mathbb{Z}}$ //

4° $n = 6k+3$: ... //

5° $n = 6k+4$: ... //

6° $n = 6k+5$: ... //

$1^\circ - 6^\circ \Rightarrow$ väite