

$$\vec{AC} = \vec{AB} + \vec{BC} = (3\vec{i} - 6\vec{j} + 6\vec{k}) + (6\vec{i} - 8\vec{k}) = 9\vec{i} - 6\vec{j} - 2\vec{k}$$

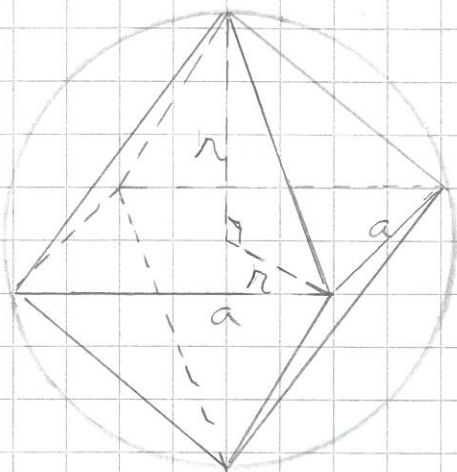
$$B = (-1+3, 1-6, 0+6) = (2, -5, 6)$$

$$C = (-1+9, 1-6, 0-2) = (8, -5, -2)$$

$$\cos(\vec{a}, \vec{AC}) = \frac{\vec{a} \cdot \vec{AC}}{|\vec{a}| |\vec{AC}|} = \frac{1 \cdot 9 + (-2) \cdot (-6) + 2 \cdot (-2)}{\sqrt{1^2 + (-2)^2 + 2^2} \sqrt{9^2 + (-6)^2 + (-2)^2}} = \frac{17}{3 \cdot 11} =$$

$$\Rightarrow \angle(\vec{a}, \vec{AC}) \approx 58,9324^\circ \approx 59^\circ$$

5.

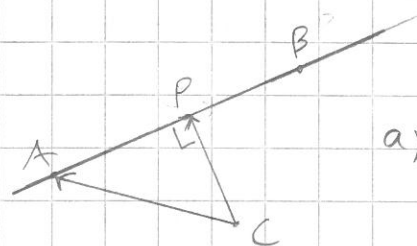


$$(2R)^2 = a^2 + a^2 \quad (\Rightarrow) 4R^2 = 2a^2 \quad (\Rightarrow) R^2 = \frac{a^2}{2}$$

$$(\Rightarrow) R = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$$

$$\frac{V_{\text{tetra}}}{V_{\text{sph}}} = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi \left(\frac{a}{\sqrt{2}}\right)^3} = \frac{\frac{4}{3}\pi \frac{a^3}{2\sqrt{2}}}{\frac{4}{3}\pi \frac{a^3}{\sqrt{2}}} = \frac{1}{2}$$

6.



$$A = (5, 0, 1), B = (1, 3, 4), C = (-1, 2, 1)$$

$$a) \vec{CP} = \vec{CA} + \vec{AP} = \vec{CA} + k\vec{AB} = (6\vec{i} - 2\vec{j}) + k(-4\vec{i} + 3\vec{j} + 3\vec{k}) = (6-4k)\vec{i} + (-2+3k)\vec{j} + 3k\vec{k}$$

$$\vec{AB} \perp \vec{CP} \quad (\Rightarrow) \vec{AB} \cdot \vec{CP} = -4 \cdot (6-4k) + 3 \cdot (-2+3k) + 3 \cdot 3k = 34k - 30 = 0 \quad (\Rightarrow) k = \frac{15}{17}$$

$$\Rightarrow \vec{CP} = \frac{42}{17}\vec{i} + \frac{11}{17}\vec{j} + \frac{45}{17}\vec{k}$$

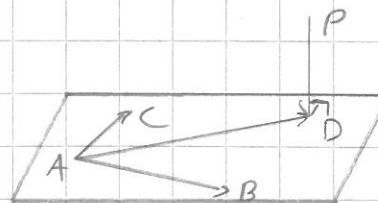
$$\Rightarrow P = (-1 + \frac{42}{17}, 2 + \frac{11}{17}, 1 + \frac{45}{17}) = (\frac{25}{17}, \frac{45}{17}, \frac{62}{17})$$

$$b) P \text{ on } \text{suoralle: } P = (-5-4k, 0+3k, 1+3k)$$

$$P \text{ on } xy\text{-tasolla: } z = 1+3k = 0 \quad (\Rightarrow) k = -\frac{1}{3}$$

$$\Rightarrow P = (5-4 \cdot (-\frac{1}{3}), 3 \cdot (-\frac{1}{3}), 1+3 \cdot (-\frac{1}{3})) = (\frac{19}{3}, -1, 0)$$

7.



$$A = (2, 1, 3), B = (3, 1, 4), C = (3, 0, 4), P = (-1, -6, 4)$$

$$\vec{PD} = \vec{PA} + \vec{AD} = \vec{PA} + \lambda \vec{AB} + k \vec{AC} = (3\vec{i} + 7\vec{j} - \vec{k}) + \lambda(\vec{i} + \vec{k}) + k(\vec{i} - \vec{j} + \vec{k}) = (3+\lambda+k)\vec{i} + (7-k)\vec{j} + (-1+\lambda+k)\vec{k}$$

$$\vec{AB} \perp \vec{PD} \quad (\Rightarrow) \vec{AB} \cdot \vec{PD} = 1 \cdot (3+\lambda+k) + 0 \cdot (7-k) + 1 \cdot (-1+\lambda+k) = 0$$

$$\vec{AC} \perp \vec{PD} \quad (\Rightarrow) \vec{AC} \cdot \vec{PD} = 1 \cdot (3+\lambda+k) + (-1) \cdot (7-k) + 1 \cdot (-1+\lambda+k) = 0$$

$$(\Rightarrow) \begin{cases} 2\lambda + 2k + 2 = 0 \\ 2\lambda + 3k - 5 = 0 \end{cases} \quad (\Rightarrow) \lambda = -8, k = 7$$

$$\Rightarrow \vec{PD} = 2\vec{i} - 2\vec{k}$$

$$\Rightarrow |\vec{PD}| = \sqrt{2^2 + (-2)^2} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

$$\Gamma_{\text{TAI}}: \vec{AB} = \vec{i} + \vec{k}, \vec{AC} = \vec{i} - \vec{j} + \vec{k}, \vec{AP} = -3\vec{i} - 7\vec{j} + \vec{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} \vec{k} = \vec{i} - \vec{k}$$

$$\text{normaali tulo: } V_h = |(\vec{AB} \times \vec{AC}) \cdot \vec{AP}| = |1 \cdot (-3) + 0 \cdot (-7) + (-1) \cdot 1| = |-4| = 4$$

$$\text{pohjan ala: } A_p = |\vec{AB} \times \vec{AC}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

$$V_h = A_p \cdot h \quad (\Rightarrow) h = \frac{V_h}{A_p} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

7.

$$x + 2y + 3z = 6$$

$$(x, 0, 0): x = 6 \quad (\Rightarrow) A = (6, 0, 0)$$

$$(0, y, 0): 2y = 6 \quad (\Rightarrow) y = 3 \quad (\Rightarrow) B = (0, 3, 0)$$

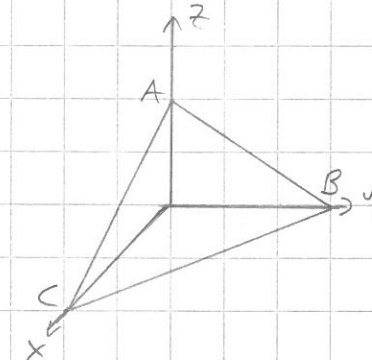
$$(0, 0, z): 3z = 6 \quad (\Rightarrow) z = 2 \quad (\Rightarrow) C = (0, 0, 2)$$

$$a) V = \frac{1}{3} A_p \cdot h = \frac{1}{3} \cdot \frac{1}{2} \cdot 6 \cdot 3 \cdot 2 = 6$$

$$b) \vec{AB} = -6\vec{i} + 3\vec{j}, \vec{AC} = -6\vec{i} + 2\vec{k}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -6 & 3 & 0 \\ -6 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} \vec{i} - \begin{vmatrix} -6 & 0 \\ -6 & 2 \end{vmatrix} \vec{j} + \begin{vmatrix} -6 & 3 \\ -6 & 0 \end{vmatrix} \vec{k} = 6\vec{i} + 12\vec{j} + 18\vec{k}$$

$$\text{normaali pituus-ala } A_k = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} \sqrt{6^2 + 12^2 + 18^2} = \frac{1}{2} \sqrt{504} = 3\sqrt{14}$$



8.

$$\phi(x, y) = x e^y - e^y - x^2$$

$$a) \begin{cases} \phi'_x(x, y) = e^y - 2x = 0 \quad (\Rightarrow) e^y = 2x \\ \phi'_y(x, y) = x e^y - e^y = 0 \quad (\Rightarrow) x \cdot 2x - 2x = 0 \quad (\Rightarrow) 2x(x-1) = 0 \end{cases}$$

$$(\Rightarrow) x = 0 \text{ tai } x = 1: \underset{>0}{e^y} = 0 \text{ y } \text{ tai } e^y = 2 \quad (\ln \Rightarrow) y = \ln 2 \quad (\Rightarrow) (1, \ln 2)$$

Kunne: piste $(1, \ln 2)$ on satulapiste

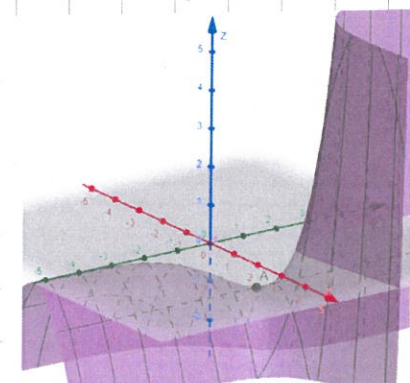
$$b) \phi'_x(2, 0) = e^0 - 2 \cdot 2 = 1 - 4 = -3$$

$$\phi'_y(2, 0) = 2e^0 - e^0 = 2 - 1 = 1$$

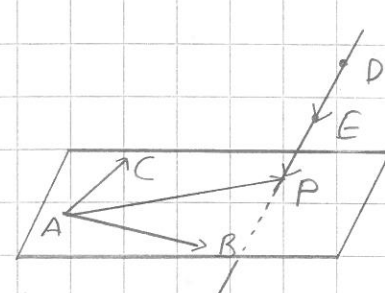
$\Rightarrow$ suurin karvennojen:

$$\nabla \phi(2, 0) = \phi'_x(2, 0)\vec{i} + \phi'_y(2, 0)\vec{j} = -3\vec{i} + \vec{j}$$

$$\text{suurin karvennojen } |\nabla \phi(2, 0)| = \sqrt{(-3)^2 + 1^2} = \sqrt{10}$$



9.



$$A = (-7, 7, 2), B = (1, -3, 1), C = (3, -5, 1)$$

$$D = (8, 7, -8), E = (7, 5, -6)$$

$$a) P \text{ on } \text{suoralla: } \vec{DP} = k \vec{DE} = k(-\vec{i} - 2\vec{j} + 3\vec{k})$$

$$\Rightarrow P = (8-k, 7-2k, -8+3k)$$

$$P \text{ on } \text{tasolla: } \vec{AP} = \lambda \vec{AB} + \mu \vec{AC}$$

$$\vec{AP} = \lambda(8\vec{i} - 10\vec{j} - \vec{k}) + \mu(10\vec{i} - 12\vec{j} - \vec{k}) = (8\lambda + 10\mu)\vec{i} + (-10\lambda - 12\mu)\vec{j} + (-\lambda - \mu)\vec{k}$$

$$\Rightarrow P = (-7 + 8\lambda + 10\mu, 7 - 10\lambda - 12\mu, 2 - \lambda - \mu)$$

Smoran jo t\u00f6n leikkauksista: