

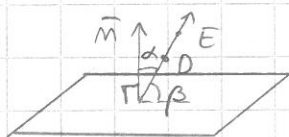
$$\begin{cases} 8-t = -7+8r+10s \\ 7-2t = 7-10r-12s \\ -8+2t = 2-r-s \end{cases} \Rightarrow t=5, r=-5, s=5 \text{ (kolmiella)}$$

$$\Rightarrow P = (8-5, 7-2\cdot 5, -8+2\cdot 5) = (3, -3, 2)$$

b) Tason normaali-vektori: $\vec{m} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -10 & -1 \\ 10 & -12 & -1 \end{vmatrix} = -2\hat{i} - 2\hat{j} + 4\hat{k}$

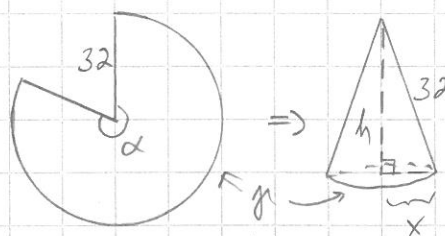
$$\cos(\vec{m}, \vec{DE}) = \frac{\vec{m} \cdot \vec{DE}}{|\vec{m}| |\vec{DE}|} = \frac{-2\cdot(-1) + (-2)\cdot(-2) + 4\cdot 2}{\sqrt{(-2)^2 + (-2)^2 + 4^2} \sqrt{(-1)^2 + (-2)^2 + 2^2}} = \frac{14}{\sqrt{24} \cdot 3}$$

$$\Rightarrow \angle(\vec{m}, \vec{DE}) = \alpha \approx 72,72^\circ$$



$$\Rightarrow \beta = 90^\circ - \alpha \approx 72,28^\circ \approx 72^\circ$$

9.



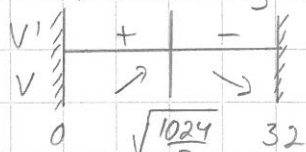
$$x^2 + h^2 = 32^2 \Rightarrow x^2 = 1024 - h^2$$

Kartion tilavuus:

$$V(h) = \frac{1}{3} \pi x^2 h = \frac{1}{3} \pi (1024 - h^2) h = \frac{\pi}{3} (1024h - h^3)$$

V on jatkuvasti derivoituva välillä $[0, 32]$

$$V'(h) = \frac{\pi}{3} (1024 - 3h^2) = 0 \Rightarrow h^2 = \frac{1024}{3} \Rightarrow h = \pm \sqrt{\frac{1024}{3}} \approx 18,425$$



$$x = \sqrt{1024 - \frac{1024}{3}} \approx 26,128$$

$$\theta = \frac{360^\circ}{2\pi \cdot 32} \cdot 2\pi x \Rightarrow \alpha = \frac{x}{32} \cdot 360^\circ \approx \frac{26,128}{32} \cdot 360^\circ \approx 293,939^\circ$$

Vast. Kartion pohjajyrmyrjän säde $x \approx 26,1$ cm, kartion korkeus: $h \approx 18,5$ cm
poistettavan ympyräsektorin keskuskulma: $360^\circ - \alpha \approx 66,1^\circ$

10. $f: \mathbb{R}^2 \rightarrow \mathbb{R}, f(x, y) = x^2 - 3xy + 3y^2$

$$\begin{aligned} a) f(x, y) &= x^2 - 3xy + 3y^2 = \left(x^2 - 2 \cdot x \cdot \frac{3}{2}y + \left(\frac{3}{2}y\right)^2\right) - \left(\frac{3}{2}y\right)^2 + 3y^2 \\ &= \left(x - \frac{3}{2}y\right)^2 + \frac{3}{4}y^2 \geq 0 \text{ aina} \end{aligned}$$

b) Olisoon $a \geq 0$ mielivaltaisen

$$f(x, y) = \left(x - \frac{3}{2}y\right)^2 + \frac{3}{4}y^2 = a$$

$$\text{Valitaan } x = \sqrt{a}, y = 0 \Rightarrow f(x, y) = f(\sqrt{a}, 0) = (\sqrt{a})^2 = a$$

$\Rightarrow f$ saa kaikki ei-negatiiviset reaalilukuarvot

MAA10 30.9.2026

1. $A = (5, -2, 1), B = (4, 3, 2), \vec{AC} = 3\hat{i} - 4\hat{j} + \hat{k}, \vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$

a) $\vec{AB} = (4-5)\hat{i} + (3-(-2))\hat{j} + (2-1)\hat{k} = -\hat{i} + 5\hat{j} + \hat{k}$
 $|\vec{AB}| = \sqrt{(-1)^2 + 5^2 + 1^2} = \sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$

b) $C = (5+3, -2-4, 1+1) = (8, -6, 2)$

c) $\vec{AC} \cdot \vec{a} = 3 \cdot 3 + (-4) \cdot 2 + 1 \cdot 1 = 2 \neq 0 \Rightarrow \vec{AB} \not\perp \vec{a}$

2.

$A = (6, 0, 2), B = (9, 2, 1), C = (11, -2, -1)$

$\vec{AB} = 3\hat{i} + 2\hat{j} - \hat{k} \Rightarrow |\vec{AB}| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{14}$

$\vec{AC} = 5\hat{i} - 2\hat{j} - 3\hat{k} \Rightarrow |\vec{AC}| = \sqrt{5^2 + (-2)^2 + (-3)^2} = \sqrt{38}$

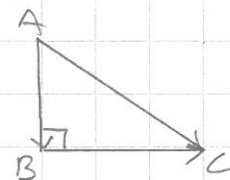
$\vec{BC} = 2\hat{i} - 4\hat{j} - 2\hat{k} \Rightarrow |\vec{BC}| = \sqrt{2^2 + (-4)^2 + (-2)^2} = \sqrt{24}$

$\vec{AB} \cdot \vec{BC} = 3 \cdot 2 + 2 \cdot (-4) + (-1) \cdot (-2) = 0 \Rightarrow \vec{AB} \perp \vec{BC} \Rightarrow \angle B = 90^\circ$

Kolmion pinta-ala: $A_k = \frac{1}{2} \cdot |\vec{AB}| |\vec{BC}| = \frac{1}{2} \cdot \sqrt{14} \sqrt{24} = \frac{1}{2} \sqrt{2 \cdot 7} \sqrt{8 \cdot 3} = 2\sqrt{21}$

TAH: $|\vec{AC}|^2 = |\vec{AB}|^2 + |\vec{BC}|^2 \Rightarrow (\sqrt{38})^2 = (\sqrt{14})^2 + (\sqrt{24})^2 \Rightarrow 38 = 14 + 24$

$\Rightarrow$ Pythagoraan lause toteutuu kolmiolla $\Rightarrow$ kolmio on suorakulmainen



3.

a) $A = (1, 2, 4), B = (2, 4, 0), C = (4, 8, -8)$

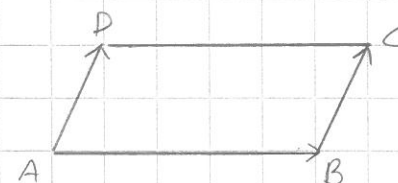
$\vec{AB} = \hat{i} + 2\hat{j} - 4\hat{k}$
 $\vec{AC} = 3\hat{i} + 6\hat{j} - 12\hat{k} \Rightarrow \vec{AC} = 3\vec{AB} \Rightarrow \vec{AC} \parallel \vec{AB} \Rightarrow$ pisteet ovat samalla suoralla

b) $f(x, y) = 5xy^2 - 2x^3 + 4y$

$f'_x(x, y) = 5y^2 - 6x^2 \Rightarrow f'_x(2, 1) = 5 \cdot 1^2 - 6 \cdot 2^2 = 5 - 24 = -19$

$f'_y(x, y) = 10xy + 4 \Rightarrow f'_y(2, 1) = 10 \cdot 2 \cdot 1 + 4 = 24$

4.



$A = (1, 1, 5), B = (2, 1, 0), C = (1, 0, 3)$

a) $\vec{AD} = \vec{BC} = (1-2)\hat{i} + (0-1)\hat{j} + (3-0)\hat{k} = -\hat{i} - \hat{j} + 3\hat{k}$

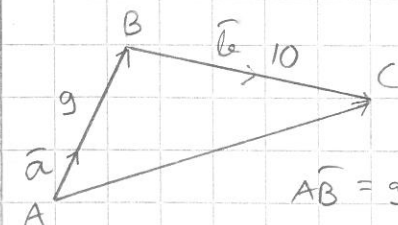
$\Rightarrow D = (1-1, 1-1, 5+3) = (0, 0, 8)$

b) $\vec{AB} = (2-1)\hat{i} + (1-1)\hat{j} + (0-5)\hat{k} = \hat{i} - 5\hat{k}$

$$\begin{aligned} \vec{AB} \times \vec{AD} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -5 \\ -1 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 0 & -5 \\ -1 & 3 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & -5 \\ -1 & 3 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} \hat{k} \\ &= (0 \cdot 3 - (-1) \cdot (-5))\hat{i} - (1 \cdot 3 - (-1) \cdot (-5))\hat{j} + (1 \cdot (-1) - (-1) \cdot 0)\hat{k} = -5\hat{i} + 2\hat{j} - \hat{k} \end{aligned}$$

Summakaavan pinta-ala: $A_k = |\vec{AB} \times \vec{AD}| = \sqrt{(-5)^2 + 2^2 + (-1)^2} = \sqrt{30}$

5.



$A = (-1, 1, 0), \vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 4\hat{k}$

$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{9} = 3$

$|\vec{b}| = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

$\vec{AB} = 9\vec{a} = 9 \frac{\vec{a}}{|\vec{a}|} = 9 \frac{\vec{a}}{3} = 3\vec{a} = 3(\hat{i} - 2\hat{j} + 2\hat{k}) = 3\hat{i} - 6\hat{j} + 6\hat{k}$

$\vec{BC} = 10\vec{b} = 10 \frac{\vec{b}}{|\vec{b}|} = 10 \frac{\vec{b}}{5} = 2\vec{b} = 2(3\hat{i} - 4\hat{k}) = 6\hat{i} - 8\hat{k}$