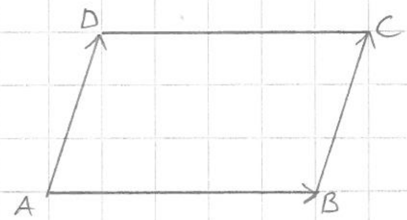
$\Rightarrow \overline{AC} = 5\bar{u} + 2\bar{v} = 5(-\bar{i} + 2\bar{j} + 2\bar{k}) + 2(-2\bar{i} - 6\bar{j} + 3\bar{k}) = -9\bar{i} - 2\bar{j} + 16\bar{k}$

$\Rightarrow C = (6-9, -8-2, 35+16) = \underline{(-3, -10, 51)}$

b) $|\overline{AC}| = \sqrt{(-9)^2 + (-2)^2 + 16^2} = \sqrt{341} \approx 18{,}5\ (cm)$

6.



$A = (1,1,5)$, $B = (2,1,0)$, $C = (1,0,3)$

a) $\overline{AD} = \overline{BC} = -\bar{i} - \bar{j} + 3\bar{k}$

$\Rightarrow D = (1-1, 1-1, 5+3) = \underline{(0,0,8)}$

b) $\overline{AB} = \bar{i} - 5\bar{k}$

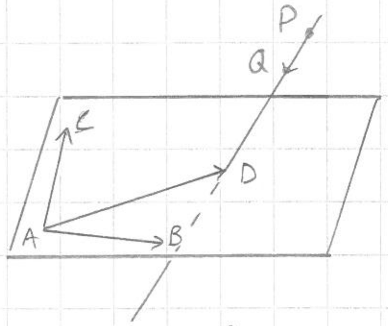
$$\overline{AB} \times \overline{AD} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 0 & -5 \\ -1 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 0 & -5 \\ -1 & 3 \end{vmatrix} \bar{i} - \begin{vmatrix} 1 & -5 \\ -1 & 3 \end{vmatrix} \bar{j} + \begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix} \bar{k}$$

$$= (0\cdot 3 - (-5)\cdot(-1))\bar{i} - (1\cdot 3 - (-5)\cdot(-1))\bar{j} + (1\cdot(-1) - (-1)\cdot 0)\bar{k} = -5\bar{i} + 2\bar{j} - \bar{k}$$

Suunnikkaan pinta-ala:

$A_s = |\overline{AB} \times \overline{AD}| = \sqrt{(-5)^2 + 2^2 + (-1)^2} = \underline{\sqrt{30}}$

7.



$A = (-9, 11, 1)$, $B = (-1, 1, 0)$, $C = (1, -1, 0)$

$P = (6, 11, -9)$, $Q = (5, 9, -7)$

1° D on suoralla $\Leftrightarrow \overline{PD} \parallel \overline{PQ}$

$\Leftrightarrow \overline{PD} = t\overline{PQ} = t(-\bar{i} - 2\bar{j} + 2\bar{k}) = -t\bar{i} - 2t\bar{j} + 2t\bar{k}$

$\Rightarrow D = (6-t, 11-2t, -9+2t)$

2° D on tasossa $\Leftrightarrow \overline{AD} = s\overline{AB} + r\overline{AC} = s(8\bar{i} - 10\bar{j} - \bar{k}) + r(10\bar{i} - 12\bar{j} - \bar{k})$

$t=5$ $\quad = (8s + 10r)\bar{i} + (-10s - 12r)\bar{j} + (-s - r)\bar{k}$

$\Rightarrow D = (-9 + 8s + 10r, 11 - 10s - 12r, 1 - s - r)$

1° ja 2° $\Rightarrow \begin{cases} 6 - t = -9 + 8s + 10r & (1) \\ 11 - 2t = 11 - 10s - 12r & (2) \\ -9 + 2t = 1 - s - r & (3) \end{cases}$

$2\cdot(1) + (3): \begin{cases} 3 = -17 + 15s + 19r & |\cdot 11 \end{cases}$

$(2) + (3): \begin{cases} 2 = 12 - 11s - 13r & |\cdot 15 \end{cases}$

$63 = -7 \quad + 14r \Leftrightarrow r = 5$

(1): $6 - t = -9 + 8\cdot(-5) + 10\cdot 5 \Leftrightarrow t = 5$

$\Rightarrow 2 = 12 - 11s - 13\cdot 5 \Leftrightarrow s = -5$

$\Rightarrow D = (6-5, 11 - 2\cdot 5, -9 + 2\cdot 5) = \underline{(1,1,1)}$

7.' $f(x,y) = 2x^2y - xy^2 - 3x + 5$

a) $f(1,2) = 2\cdot 1^2 \cdot 2 - 1\cdot 2^2 - 3\cdot 1 + 5 = \underline{2}$

b) $f'_x(x,y) = 4xy - y^2 - 3 \Rightarrow f'_x(1,2) = 4\cdot 1\cdot 2 - 2^2 - 3 = 1$

$\Rightarrow$ nousee kulmakertoimella 1 eli 45° kulmassa

c) $f'_y(x,y) = 2x^2 - 2xy \Rightarrow f'_y(1,2) = 2\cdot 1^2 - 2\cdot 1\cdot 2 = -2$

$\nabla f(1,2) = f'_x(1,2)\bar{i} + f'_y(1,2)\bar{j} = \bar{i} - 2\bar{j}$

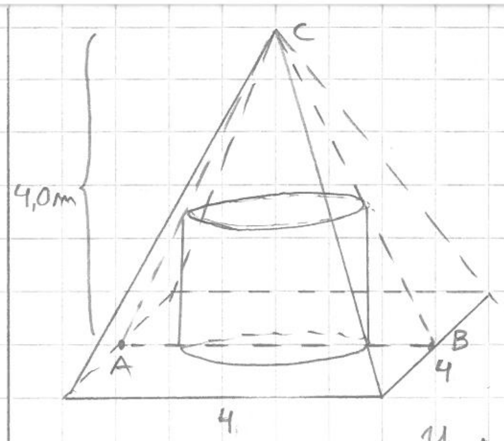
$\Rightarrow$ suurin nousu suuntaan $\bar{i} - 2\bar{j}$ ja kulmakerroin $|\nabla f(1,2)| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

$\Rightarrow$ murikka liukuu alaspäin suuntaan $-(\bar{i} - 2\bar{j}) = -\bar{i} + 2\bar{j}$

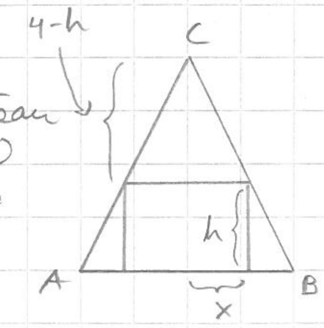
kulmakerroin: $k = \tan\alpha = \sqrt{5} \Rightarrow \alpha \approx 65{,}905°$

$\Rightarrow a = 9{,}81\ \frac{m}{s^2} \cdot \sin 65{,}905° \approx 8{,}955\ \frac{m}{s^2} \approx \underline{9{,}0\ \frac{m}{s^2}}$

8.



leikataan $\Rightarrow$ tasolla



$\triangle \sim \triangle$ (kk)

$\frac{4-h}{2x} = \frac{4}{4} \quad |\cdot 2x$

$\Leftrightarrow 4 - h = 2x$

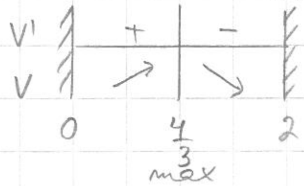
$\Leftrightarrow h = 4 - 2x$

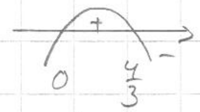
Ympyrälieriön tilavuus:

$V(x) = \pi x^2 h = \pi x^2(4 - 2x) = 4\pi x^2 - 2\pi x^3$

V on jatk. ja deriv. välillä $[0, 2]$

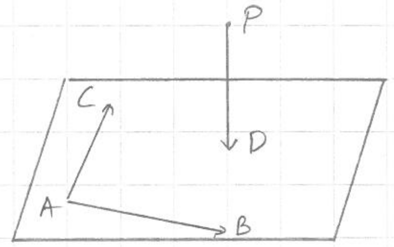
$V'(x) = 8\pi x - 6\pi x^2 = 2\pi x(4 - 3x) = 0 \Leftrightarrow x = 0$ tai $x = \frac{4}{3}$





suurin arvo: $V(\frac{4}{3}) = 4\pi(\frac{4}{3})^2 - 2\pi(\frac{4}{3})^3 = \underline{\frac{64\pi}{27}}$

9.



$A = (1, -1, 2)$, $B = (2, 0, 1)$, $C = (-1, 2, 3)$, $P = (6, 8, 13)$

D on tasolla $\Leftrightarrow \overline{AD} = s\overline{AB} + t\overline{AC}$

$\Leftrightarrow \overline{AD} = s(\bar{i} + \bar{j} - \bar{k}) + t(-2\bar{i} + 3\bar{j} + \bar{k}) = (s - 2t)\bar{i} + (s + 3t)\bar{j} + (-s + t)\bar{k}$

$\Rightarrow D = (1 + s - 2t, -1 + s + 3t, 2 - s + t)$

a) D on x-akselilla $\Leftrightarrow \begin{cases} y = -1 + s + 3t = 0 \\ z = 2 - s + t = 0 \end{cases} \Leftrightarrow s = \frac{7}{4}, t = -\frac{1}{4} \Rightarrow \underline{D = (\frac{13}{4}, 0, 0)}$

D on y-akselilla $\Leftrightarrow \begin{cases} x = 1 + s - 2t = 0 \\ z = 2 - s + t = 0 \end{cases} \Leftrightarrow t = 3, s = 5 \Rightarrow \underline{D = (0, 13, 0)}$

D on z-akselilla $\Leftrightarrow \begin{cases} x = 1 + s - 2t = 0 \\ y = -1 + s + 3t = 0 \end{cases} \Leftrightarrow t = \frac{2}{5}, s = -\frac{1}{5} \Rightarrow \underline{D = (0, 0, \frac{13}{5})}$

b) $\overline{PD} = (-5 + s - 2t)\bar{i} + (-9 + s + 3t)\bar{j} + (-11 - s + t)\bar{k}$

$\begin{cases} \overline{PD} \perp \overline{AB} \Leftrightarrow \overline{PD}\cdot\overline{AB} = (-5 + s - 2t)\cdot 1 + (-9 + s + 3t)\cdot 1 + (-11 - s + t)\cdot(-1) = 0 \\ \overline{PD} \perp \overline{AC} \Leftrightarrow \overline{PD}\cdot\overline{AC} = (-5 + s - 2t)\cdot(-2) + (-9 + s + 3t)\cdot 3 + (-11 - s + t)\cdot 1 = 0 \end{cases}$

$\Leftrightarrow \begin{cases} 3s - 3 = 0 & \Leftrightarrow s = 1 \\ 14t - 28 = 0 & \Leftrightarrow t = 2 \end{cases} \Rightarrow \overline{PD} = -8\bar{i} - 2\bar{j} - 10\bar{k}$

$\Rightarrow |\overline{PD}| = \sqrt{(-8)^2 + (-2)^2 + (-10)^2} = \sqrt{168} = \underline{2\sqrt{42}}\ (\approx 12{,}96)$

TAI:

$$\bar{n} = \overline{AB} \times \overline{AC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & -1 \\ -2 & 3 & 1 \end{vmatrix} = 4\bar{i} + \bar{j} + 5\bar{k}$$ (tason normaalivektori)

$\Rightarrow$ taso: $4x + y + 5z + d = 0$

$A = (1, -1, 2)$ on tasolla: $4\cdot 1 + (-1) + 5\cdot 2 + d = 0 \Leftrightarrow d = -13$

$\Rightarrow 4x + y + 5z - 13 = 0$ (tason normaalimuoto)

a) $4x - 13 = 0 \Leftrightarrow x = \frac{13}{4} \Rightarrow (\frac{13}{4}, 0, 0)$; $y - 13 = 0 \Leftrightarrow y = 13 \Rightarrow \underline{(0, 13, 0)}$

$5z - 13 = 0 \Leftrightarrow z = \frac{13}{5} \Rightarrow \underline{(0, 0, \frac{13}{5})}$