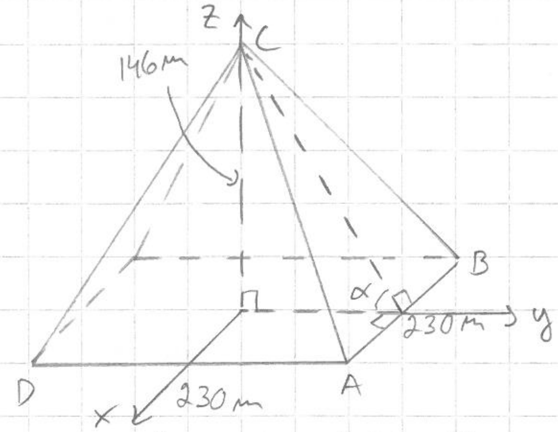
$\overline{PD} \parallel \bar{n} \Leftrightarrow \overline{PD} = t\bar{n} = 4t\bar{i} + t\bar{j} + 5t\bar{k} \Rightarrow D = (6+4t, 8+t, 13+5t)$

D on tasolla: $4(6+4t) + (8+t) + 5(13+5t) - 13 = 0 \Leftrightarrow 42t + 84 = 0 \Leftrightarrow t = -2$

$\Rightarrow \overline{PD} = -8\bar{i} - 2\bar{j} - 10\bar{k} \Rightarrow |\overline{PD}| = \sqrt{(-8)^2 + (-2)^2 + (-10)^2} = \underline{2\sqrt{42}}$

10.



a) $\tan\alpha = \dfrac{146\,m}{\frac{1}{2}\cdot 230\,m} \Rightarrow \alpha \approx 51{,}774° \approx \underline{52°}$

b) $A = (115, 115, 0)$, $B = (-115, 115, 0)$

$C = (0, 0, 146)$, $D = (115, -115, 0)$

$\overline{AB} = -230\bar{i}$ $\quad \overline{AC} = -115\bar{i} - 115\bar{j} + 146\bar{k}$

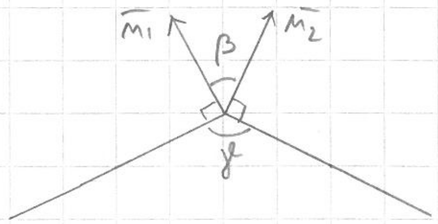
$\overline{AD} = -230\bar{j}$

Lasketaan sivutahkoille ABC ja ADC yhteisiin tasoon sojottavat normaalivektorit:

$$\bar{n}_1 = \overline{AB} \times \overline{AC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -230 & 0 & 0 \\ -115 & -115 & 146 \end{vmatrix} = 33580\bar{j} + 26450\bar{k}$$

$$\bar{n}_2 = \overline{AC} \times \overline{AD} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -115 & -115 & 146 \\ 0 & -230 & 0 \end{vmatrix} = 33580\bar{i} + 26450\bar{k}$$

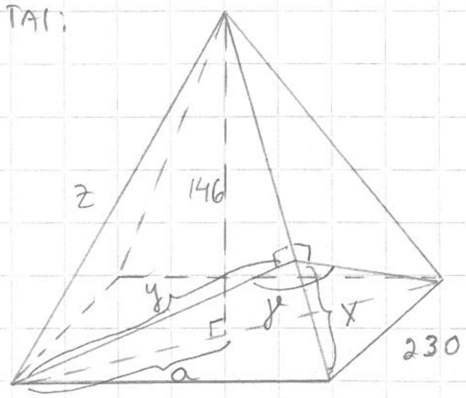
$$\cos(\bar{n}_1, \bar{n}_2) = \frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1||\bar{n}_2|} = \frac{26450^2}{33580^2 + 26450^2} \Rightarrow \sphericalangle(\bar{n}_1, \bar{n}_2) = \beta \approx 67{,}488°$$



Sivutahkojen välinen kulma on pyramidin sisällä eli

$\gamma = 360° - 90° - 90° - \beta \approx 112{,}512° \approx \underline{113°}$

TAI:



Pythagoras: $230^2 + 230^2 = (2a)^2 \quad |\sqrt{\ } \quad |:2$

$\Leftrightarrow a = \sqrt{2}\cdot 115$

$$\begin{cases} a^2 + 146^2 = z^2 \\ x^2 + y^2 = 230^2 \\ y^2 + (z-x)^2 = z^2 \end{cases} \Leftrightarrow z = \pm\sqrt{2\cdot 115^2 + 146^2} = \sqrt{47766}$$

$\Rightarrow 230^2 - x^2 + z^2 - 2zx + x^2 = z^2$

$\Leftrightarrow x = \dfrac{230^2}{2z} = \dfrac{230^2}{2\sqrt{47766}} = \dfrac{26450}{\sqrt{47766}}$

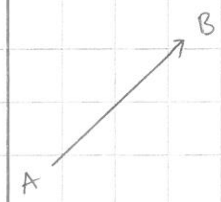
$y^2 = 230^2 - x^2 = 230^2 - \dfrac{26450^2}{47766} = \dfrac{913609450}{23883}$

Kosinilause: $(2a)^2 = y^2 + y^2 - 2\cdot y \cdot y \cdot \cos\gamma$

$\Leftrightarrow \cos\gamma = \dfrac{(2\cdot\sqrt{2}\cdot 115)^2 - 2y^2}{-2y^2} = -\dfrac{13225}{34541} \Rightarrow \gamma \approx 112{,}512° \approx \underline{113°}$

MAA10 27.9.2023

1.



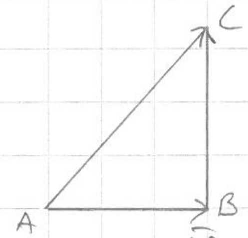
$A = (-1, 3, 2)$, $\overline{AB} = 2\bar{i} - \bar{j} + 3\bar{k}$, $\bar{n} = 5\bar{i} + 12\bar{j} + \bar{k}$

a) $B = (-1+2, 3-1, 2+3) = \underline{(1, 2, 5)}$

b) $|\overline{AB}| = \sqrt{2^2 + (-1)^2 + 3^2} = \underline{\sqrt{14}}$

c) $\overline{AB}\cdot\bar{n} = 2\cdot 5 + (-1)\cdot 12 + 3\cdot 1 = 1 \neq 0 \Rightarrow$ ei ole

2.



$A = (3, 1, 0)$, $B = (6, 3, -1)$, $C = (8, -1, -3)$

$\overline{AB} = 3\bar{i} + 2\bar{j} - \bar{k} \Rightarrow |\overline{AB}| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{14}$

$\overline{AC} = 5\bar{i} - 2\bar{j} - 3\bar{k} \Rightarrow |\overline{AC}| = \sqrt{5^2 + (-2)^2 + (-3)^2} = \sqrt{38}$

$\overline{BC} = 2\bar{i} - 4\bar{j} - 2\bar{k} \Rightarrow |\overline{BC}| = \sqrt{2^2 + (-4)^2 + (-2)^2} = \sqrt{24}$

$\overline{AB}\cdot\overline{BC} = 3\cdot 2 + 2\cdot(-4) + (-1)\cdot(-2) = 6 - 8 + 2 = 0 \Rightarrow \overline{AB} \perp \overline{BC} \Rightarrow \sphericalangle B = 90°$

Pinta-ala: $A = \frac{1}{2}|\overline{AB}||\overline{BC}| = \frac{1}{2}\cdot\sqrt{14}\cdot\sqrt{24} = \frac{1}{2}\sqrt{2}\sqrt{7}\cdot\sqrt{2}\sqrt{4}\cdot\sqrt{3} = \underline{2\sqrt{21}}$

3. Suora: $\begin{cases} x = 2 + t \\ y = -1 - 2t \quad (t \in \mathbb{R}) \\ z = 3 - t \end{cases}$ $\qquad$ Taso: $x - 3y + 2z + 4 = 0$

a) $(2+t) - 3(-1-2t) + 2(3-t) + 4 = 0 \Leftrightarrow 5t + 15 = 0 \Leftrightarrow t = -3$

$\Rightarrow$ leikkauspiste: $(2-3, -1-2\cdot(-3), 3-(-3)) = \underline{(-1, 5, 6)}$

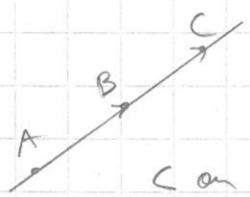
b) Piste on yz-tasolla $\Leftrightarrow x = 2 + t = 0 \Leftrightarrow t = -2$

$\Rightarrow$ piste: $(2-2, -1-2\cdot(-2), 3-(-2)) = \underline{(0, 3, 5)}$

c) Piste on z-akselilla $\Leftrightarrow x = y = 0 \Rightarrow 2z + 4 = 0 \Leftrightarrow z = -2$

$\Rightarrow$ piste: $\underline{(0, 0, -2)}$

4.



$A = (40, -10, 50)$, $B = (0, 10, 30)$, $C = (120, -50, 110)$

a) $\overline{AB} = -40\bar{i} + 20\bar{j} - 20\bar{k}$

$\overline{AC} = 80\bar{i} - 40\bar{j} + 60\bar{k}$

C on suoralla $\Leftrightarrow \overline{AB} \parallel \overline{AC} \Leftrightarrow \overline{AC} = t\overline{AB} \Leftrightarrow 80\bar{i} - 40\bar{j} + 60\bar{k} = -40t\bar{i} + 20t\bar{j} - 20t\bar{k}$

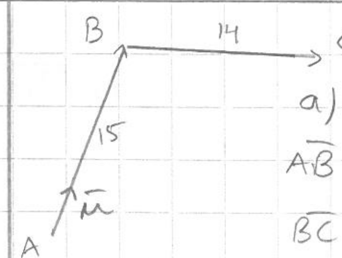
$$\Rightarrow \begin{cases} 80 = -40t & \Leftrightarrow t = -2 \\ -40 = 20t & \Leftrightarrow t = -2 \\ 60 = -20t & \Leftrightarrow t = -3 \end{cases} \text{ y } \Rightarrow C \text{ ei ole suoralla}$$

b) P on suoralla $\Leftrightarrow \overline{AP} \parallel \overline{AB} \Leftrightarrow \overline{AP} = t\overline{AB} = -40t\bar{i} + 20t\bar{j} - 20t\bar{k}$

$\Rightarrow P = (40 - 40t, -10 + 20t, 50 - 20t)$

P on xy-tasolla $\Leftrightarrow z = 50 - 20t = 0 \Leftrightarrow t = \frac{5}{2} \Rightarrow \underline{P = (-60, 40, 0)}$

5.



$A = (6, -8, 35)$, $\bar{u} = -\bar{i} + 2\bar{j} + 2\bar{k}$, $\bar{v} = -2\bar{i} - 6\bar{j} + 3\bar{k}$

a) $|\bar{u}| = \sqrt{(-1)^2 + 2^2 + 2^2} = \sqrt{9} = 3$, $|\bar{v}| = \sqrt{(-2)^2 + (-6)^2 + 3^2} = \sqrt{49} = 7$

$\overline{AB} = 15\bar{u}^0 = 15\cdot\dfrac{\bar{u}}{|\bar{u}|} = 15\cdot\dfrac{\bar{u}}{3} = 5\bar{u}$

$\overline{BC} = 14\bar{v}^0 = 14\cdot\dfrac{\bar{v}}{|\bar{v}|} = 14\cdot\dfrac{\bar{v}}{7} = 2\bar{v}$