MAA10 24.9.2025

1. $A = (-2, 1, 6)$, $B = (3, 0, 4)$, $\bar u = \bar i + 3\bar j + 4\bar k$

a) $\overline{AB} = (3-(-2))\bar i + (0-1)\bar j + (4-6)\bar k = \underline{5\bar i - \bar j - 2\bar k}$

$|\overline{AB}| = \sqrt{5^2 + (-1)^2 + (-2)^2} = \underline{\sqrt{30}}$

b) $\overline{AB}\cdot\bar u = 5\cdot 1 + (-1)\cdot 3 + (-2)\cdot 4 = 5 - 3 - 8 = -6 \neq 0 \Rightarrow \underline{\overline{AB} \not\perp \bar u}$

2. a) $\bar a = 2\bar i + \bar j - 2\bar k$, $\bar b = x^2\bar i + 2x\bar j + 2\bar k$

$\bar a \perp \bar b \Leftrightarrow \bar a\cdot\bar b = 2\cdot x^2 + 1\cdot 2x + (-2)\cdot 2 = 2x^2 + 2x - 4 = 0$

$\Leftrightarrow \underline{x} = \dfrac{-2 \pm\sqrt{2^2 - 4\cdot 2\cdot(-4)}}{2\cdot 2} = \dfrac{-2\pm\sqrt{36}}{4} = \dfrac{-2\pm 6}{4} = \underline{\begin{cases}1\\-2\end{cases}}$

b) $f(x,y) = 2xy^2 - 4x^2 + 5y$

$f'_x(x,y) = 2y^2 - 8x \Rightarrow f'_x(3,1) = 2\cdot 1^2 - 8\cdot 3 = \underline{-22}$

$f'_y(x,y) = 4xy + 5 \Rightarrow f'_y(3,1) = 4\cdot 3\cdot 1 + 5 = \underline{17}$

3. $S: \begin{cases} x = 4 + t \\ y = -4 + 2t,\ t\in\mathbb{R} \\ z = 5 + 4t \end{cases}$ $T: \ 3x + y - 5z = 3$

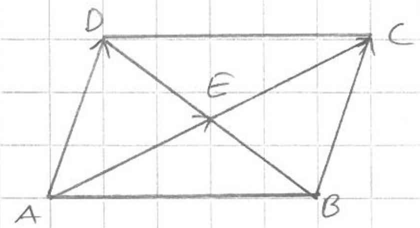
a) $P = (x, y, 0)$ on xy-tasolla: $z = 5 + 4t = 0 \Leftrightarrow t = -\frac{5}{4}$

$\Rightarrow P = (4 - \frac54, -4 + 2\cdot(-\frac54), 5 + 4\cdot(-\frac54)) = \underline{(\frac{11}{4}, -\frac{13}{2}, 0)}$

b) $P = (x, 0, 0)$ on x-akselilla: $3x = 3 \Leftrightarrow x = 1 \Rightarrow P = \underline{(1, 0, 0)}$

c) $3(4+t) + (-4+2t) - 5(5+4t) = 3 \Leftrightarrow -15t = 20 \Leftrightarrow t = -\frac43$

$\Rightarrow P = (4 - \frac43, -4 + 2\cdot(-\frac43), 5 + 4\cdot(-\frac43)) = \underline{(\frac83, -\frac{20}{3}, -\frac13)}$

4. 

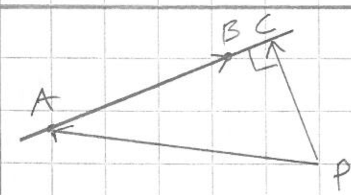
$A = (5, 2, 1)$, $B = (-1, 3, 0)$, $C = (1, -2, 4)$

$\overline{AD} = \overline{BC} = (1-(-1))\bar i + (-2-3)\bar j + (4-0)\bar k = 2\bar i - 5\bar j + 4\bar k$

$\Rightarrow \underline{D} = (5+2, 2-5, 1+4) = \underline{(7, -3, 5)}$

$\overline{AE} = \frac12\overline{AC} = \frac12(-4\bar i - 4\bar j + 3\bar k) = -2\bar i - 2\bar j + \frac32\bar k$

$\Rightarrow E = (5-2, 2-2, 1+\frac32) = \underline{(3, 0, \frac52)}$

5. 

$A = (-4, 0, 1)$, $B = (2, -3, 7)$, $P = (2, 3, 1)$

$\overline{PC} = \overline{PA} + \overline{AC} = \overline{PA} + t\overline{AB}$
$= (-6\bar i - 3\bar j) + t(6\bar i - 3\bar j + 6\bar k)$
$= (-6 + 6t)\bar i + (-3 - 3t)\bar j + 6t\bar k$

$\overline{PC} \perp \overline{AB} \Leftrightarrow \overline{PC}\cdot\overline{AB} = (-6+6t)\cdot 6 + (-3-3t)\cdot(-3) + 6t\cdot 6 = 0$

$\Leftrightarrow 81t - 27 = 0 \Leftrightarrow t = \frac13$

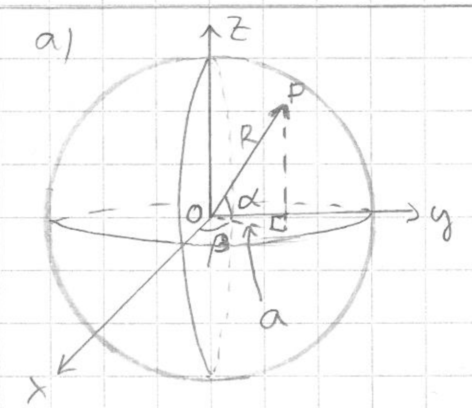
$\Rightarrow \overline{PC} = (-6 + 6\cdot\frac13)\bar i + (-3 - 3\cdot\frac13)\bar j + 6\cdot\frac13\bar k = -4\bar i - 4\bar j + 2\bar k$

TAI: $\overline{A'B'} = -4\bar i + 12\bar j$, $\overline{A'C'} = -8\bar i + 12\bar j$

$\overline{A'B'}\times\overline{A'C'} = \begin{vmatrix}\bar i & \bar j & \bar k\\ -4 & 12 & 0\\ -8 & 12 & 0\end{vmatrix} = \begin{vmatrix}12 & 0\\ 12 & 0\end{vmatrix}\bar i - \begin{vmatrix}-4 & 0\\ -8 & 0\end{vmatrix}\bar j + \begin{vmatrix}-4 & 12\\ -8 & 12\end{vmatrix}\bar k$

$= (12\cdot 0 - 12\cdot 0)\bar i - (-4\cdot 0 - (-8)\cdot 0)\bar j + (-4\cdot 12 - (-8)\cdot 12)\bar k = 48\bar k$

Kolmion pinta-ala: $A_k = \frac12|\overline{A'B'}\times\overline{A'C'}| = \frac12\cdot 48 = \underline{24}$

10. a) 

$R = 6366$ km

$\cos\alpha = \dfrac{a}{R} \Leftrightarrow a = R\cos\alpha$

$\cos\beta = \dfrac{x}{a} \Leftrightarrow \underline{x} = a\cos\beta = \underline{R\cos\alpha\cos\beta}$

$\sin\beta = \dfrac{y}{a} \Leftrightarrow \underline{y} = a\sin\beta = \underline{R\cos\alpha\sin\beta}$

$\sin\alpha = \dfrac{z}{R} \Leftrightarrow \underline{z = R\sin\alpha}$

b) $K: \alpha = 60{,}87°$, $\beta = 26{,}32°$

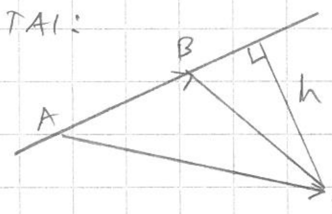
$I: \alpha_1 = 22{,}82°$, $\beta = 108{,}32°$

$\cos(\overline{OK}, \overline{OI}) = \dfrac{\overline{OK}\cdot\overline{OI}}{|\overline{OK}||\overline{OI}|} = \dfrac{R^2\cos\alpha\cos\beta\cos\alpha_1\cos\beta_1 + R^2\cos\alpha\sin\beta\cos\alpha_1\sin\beta_1 + R^2\sin\alpha\sin\alpha_1}{R^2}$

$= 0{,}4012283 \Rightarrow \sphericalangle(\overline{OK}, \overline{OI}) \approx 66{,}3450° \approx \underline{66{,}34°}$

c) Etäisyys: $s = \dfrac{\sphericalangle(\overline{OK}, \overline{OI})}{360°}\cdot 2\pi R = \dfrac{66{,}345°}{360°}\cdot 2\pi\cdot 6366$ km

$\approx 7371{,}4377$ km $\approx \underline{7370 \text{ km}}$

5. TAI: 

$A = (-4, 0, 1)$, $B = (2, -3, 7)$, $P = (2, 3, 1)$

$\overline{AB} = 6\bar i - 3\bar j + 6\bar k$ $\quad \overline{AP} = 6\bar i + 3\bar j$

$\overline{AB}\times\overline{AP} = \begin{vmatrix}\bar i & \bar j & \bar k\\ 6 & -3 & 6\\ 6 & 3 & 0\end{vmatrix} = \begin{vmatrix}-3 & 6\\ 3 & 0\end{vmatrix}\bar i - \begin{vmatrix}6 & 6\\ 6 & 0\end{vmatrix}\bar j + \begin{vmatrix}6 & -3\\ 6 & 3\end{vmatrix}\bar k$

$= (-3\cdot 0 - 3\cdot 6)\bar i - (6\cdot 0 - 6\cdot 6)\bar j + (6\cdot 3 - 6\cdot(-3))\bar k = -18\bar i + 36\bar j + 36\bar k$

$A_k = \frac12|\overline{AB}\times\overline{AP}| = \frac12\sqrt{(-18)^2 + 36^2 + 36^2} = \underline{27}$

$A_k = \frac12|\overline{AB}|\cdot h = \frac12\sqrt{6^2 + (-3)^2 + 6^2}\cdot h = \frac12\cdot 9\cdot h = 27 \Leftrightarrow h = \dfrac{2\cdot 27}{9} = \underline{6}$