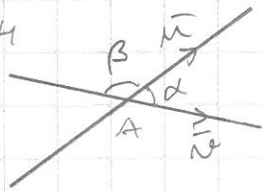


7. Suorien leikkaaminen

7.4



$$A = (5, -4, 0), \quad \vec{u} = \vec{i} - \vec{j} + 2\vec{k}$$

$$\vec{v} = 3\vec{i} + 2\vec{j} - \vec{k}$$

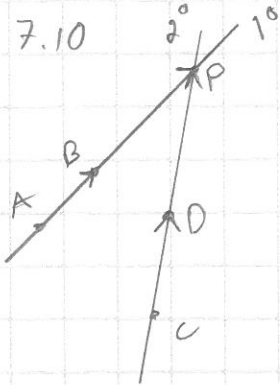
$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos(\vec{u}, \vec{v})$$

$$\Rightarrow \cos(\vec{u}, \vec{v}) = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}| |\vec{v}|} = \frac{1 \cdot 3 + (-1) \cdot 2 + 2 \cdot (-1)}{\sqrt{1^2 + (-1)^2 + 2^2} \cdot \sqrt{3^2 + 2^2 + (-1)^2}} = \frac{-1}{\sqrt{6} \sqrt{14}}$$

$$\Rightarrow \alpha = \angle(\vec{u}, \vec{v}) \approx 96,26^\circ \approx 96^\circ > 90^\circ$$

Suorien välisen kulma ($\leq 90^\circ$): $\beta = 180^\circ - \alpha = 180^\circ - 96^\circ = \underline{84^\circ}$

7.10



$$A = (2, -1, 8), \quad B = (5, -1, -7)$$

$$C = (0, -13, 3), \quad D = (2, -5, 3)$$

1° P on suora 1 $\Rightarrow \vec{AP} \parallel \vec{AB}$

$$\Rightarrow \vec{AP} = t \vec{AB} = t(3\vec{i} - 15\vec{k})$$

$$= 3t\vec{i} - 15t\vec{k}$$

$$\Rightarrow P = (2+3t, -1, 8-15t)$$

2° P on suora 2 $\Rightarrow \vec{CP} \parallel \vec{CD}$

$$\Rightarrow \vec{CP} = s \vec{CD} = s(2\vec{i} + 8\vec{j}) = 2s\vec{i} + 8s\vec{j}$$

$$\Rightarrow P = (0+2s, -13+8s, 3)$$

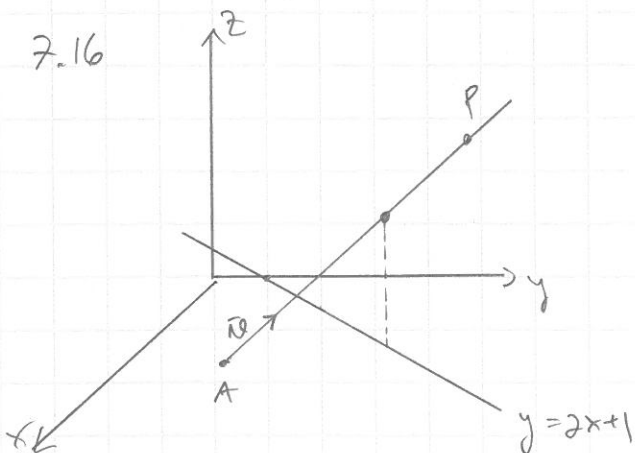
1° ja 2° $\Rightarrow$ suorat leikkaavat

$$\Rightarrow \begin{cases} 2+3t = 2s \\ -1 = -13+8s \\ 8-15t = 3 \end{cases} \Rightarrow \begin{cases} t = \frac{1}{3} \\ s = \frac{3}{2} \end{cases}$$

Tark. : $2+3 \cdot \frac{1}{3} = 2 \cdot \frac{3}{2} \Rightarrow 3 = 3$

Vast. leikkaavien piste : $P = (2+3 \cdot \frac{1}{3}, -1, 8-15 \cdot \frac{1}{3}) = \underline{(3, -1, 3)}$

7.16



$$A = (3, 1, 0), \quad \vec{u} = -\vec{i} + 3\vec{j} + \vec{k}$$

P on suoran suora $\Rightarrow \vec{AP} \parallel \vec{u}$

$$\Rightarrow \vec{AP} = t \vec{u} = -t\vec{i} + 3t\vec{j} + t\vec{k}, \quad t \geq 0$$

$$\Rightarrow P = (\underbrace{3-t}_x, \underbrace{1+3t}_y, \underbrace{0+t}_z)$$

Piste P on suoran $y = 2x + 1$ tasosuora

$$\Rightarrow 1+3t = 2 \cdot (3-t) + 1$$

$$\Rightarrow 1+3t = 6-2t+1 \Rightarrow 5t = 6 \Rightarrow t = \frac{6}{5}$$

$$\Rightarrow P = (3 - \frac{6}{5}, 1 + 3 \cdot \frac{6}{5}, \frac{6}{5}) = (\frac{9}{5}, \frac{23}{5}, \frac{6}{5}) = \underline{(1,8; 4,6; 1,2)}$$

$\Rightarrow$ korkeus: $z = \underline{1,2 \text{ km}}$