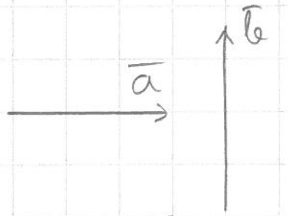
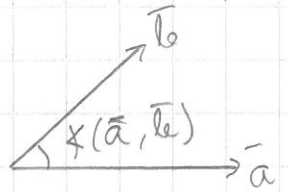


Olkoon $\bar a, \bar b \neq \bar 0$

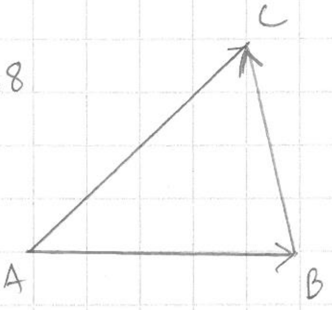
$\boxed{\bar a \perp \bar b \Leftrightarrow \bar a \cdot \bar b = 0}$ KOHTISUORUUSEHTO



$$\boxed{\bar a \cdot \bar b = |\bar a||\bar b| \cos(\bar a, \bar b)}$$

$$\Leftrightarrow \cos(\bar a, \bar b) = \frac{\bar a \cdot \bar b}{|\bar a||\bar b|}$$

3,8



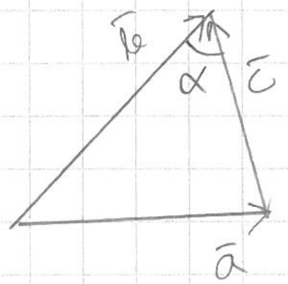
$A = (-2, t, 4)$, $B = (3, 0, -1)$, $C = (1, -2, -3)$

$\bar{AB} = 5\bar i - t\bar j - 5\bar k$
$\bar{AC} = 3\bar i + (-2-t)\bar j - 7\bar k$
$\bar{BC} = -2\bar i - 2\bar j - 2\bar k$

a) $\sphericalangle B = 90^\circ \Leftrightarrow \bar{AB} \perp \bar{BC} \Leftrightarrow \bar{AB} \cdot \bar{BC} = 5 \cdot (-2) + (-t) \cdot (-2) + (-5) \cdot (-2) = 0$
$\Leftrightarrow -10 + 2t + 10 = 0 \Leftrightarrow 2t = 0 \Leftrightarrow t = 0$

b) $\sphericalangle C = 90^\circ \Leftrightarrow \bar{AC} \perp \bar{BC} \Leftrightarrow \bar{AC} \cdot \bar{BC} = 3 \cdot (-2) + (-2-t) \cdot (-2) + (-7) \cdot (-2) = 0$
$\Leftrightarrow -6 + 4 + 2t + 14 = 0 \Leftrightarrow 2t = -12 \Leftrightarrow \underline{t = -6}$

3,9



$\bar a = 2\bar i - \bar j - 2\bar k \Rightarrow |\bar a| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3$
$\bar b = 3\bar i + \bar j + \bar k \Rightarrow |\bar b| = \sqrt{3^2 + 1^2 + 1^2} = \sqrt{11}$
$\bar c = -\bar a + \bar b = \bar i + 2\bar j + 3\bar k \Rightarrow |\bar c| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

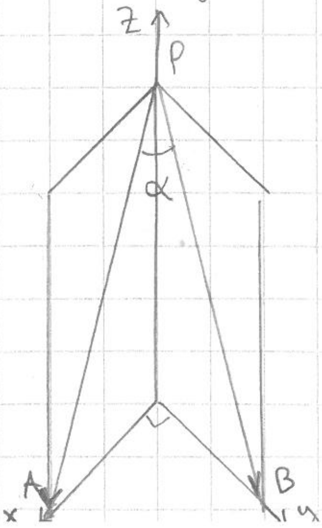
$\Rightarrow$ $\bar a$ on lyhin $\Rightarrow$ vastainen kulma α on pienin

$$\cos\alpha = \cos(-\bar b, -\bar c) = \frac{-\bar b \cdot (-\bar c)}{|-\bar b||-\bar c|} = \frac{\bar b \cdot \bar c}{|\bar b||\bar c|} = \frac{3 \cdot 1 + 1 \cdot 2 + 1 \cdot 3}{\sqrt{11} \cdot \sqrt{14}} = \frac{8}{\sqrt{11}\sqrt{14}}$$

vektorien yhteinen alkupiste α:n kärjessä

$\Rightarrow \underline{\alpha \approx 49{,}9^\circ}$

3.19



$A = (18{,}4 ; 0 ; 0)$, $B = (0 ; 18{,}4 ; 0)$
$P = (0 ; 0 ; 23{,}8)$

$\bar{PA} = 18{,}4\bar i - 23{,}8\bar k$
$\bar{PB} = 18{,}4\bar j - 23{,}8\bar k$

$$\cos\alpha = \cos(\bar{PA}, \bar{PB}) = \frac{\bar{PA} \cdot \bar{PB}}{|\bar{PA}||\bar{PB}|} = \frac{18{,}4 \cdot 0 + 0 \cdot 18{,}4 + (-23{,}8) \cdot (-23{,}8)}{\sqrt{18{,}4^2 + (-23{,}8)^2}\ \sqrt{18{,}4^2 + (-23{,}8)^2}}$$

$\Rightarrow \alpha = \sphericalangle(\bar{PA}, \bar{PB}) \approx \underline{51{,}3^\circ}$